**90 Minutes—Scientific Calculator**

Notes: (1) The <u>exact</u> numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that <u>best approximates</u> the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$

   (A) −6   (B) −3   (C) 3   (D) 6   (E) 8
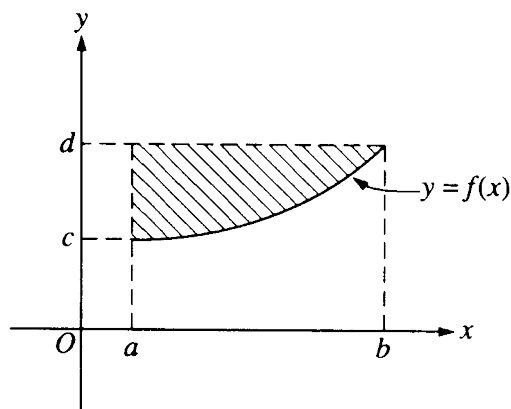
2. Which of the following represents the area of the shaded region in the figure above?

   (A) $\displaystyle\int_c^d f(y)\,dy$ (B) $\displaystyle\int_a^b \big(d - f(x)\big)\,dx$ (C) $f'(b) - f'(a)$

   (D) $(b-a)\big[f(b) - f(a)\big]$ (E) $(d-c)\big[f(b) - f(a)\big]$

3. $\displaystyle\lim_{n \to \infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

   (A) −5   (B) −2   (C) 1   (D) 3   (E) nonexistent

4.   If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\dfrac{dy}{dx} =$

(A) $-\dfrac{x^2 + y}{x + 2y^2}$

(B) $-\dfrac{x^2 + y}{x + y^2}$

(C) $-\dfrac{x^2 + y}{x + 2y}$

(D) $-\dfrac{x^2 + y}{2y^2}$

(E) $\dfrac{-x^2}{1 + 2y^2}$

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5.   If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 4}{x + 2}$ when $x \neq -2$,

then $f(-2) =$

(A) -4 (B) -2 (C) -1 (D) 0 (E) 2

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6.   The area of the region enclosed by the curve $y = \dfrac{1}{x - 1}$, the x-axis, and the lines $x = 3$ and $x = 4$ is

(A) $\dfrac{5}{36}$ (B) $\ln \dfrac{2}{3}$ (C) $\ln \dfrac{4}{3}$ (D) $\ln \dfrac{3}{2}$ (E) $\ln 6$

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7.   An equation of the line tangent to the graph of $y = \dfrac{2x + 3}{3x - 2}$ at the point $(1, 5)$ is

(A) $13x - y = 8$ (B) $13x + y = 18$ (C) $x - 13y = 64$

(D) $x + 13y = 66$ (E) $-2x + 3y = 13$

8. If $y = \tan x - \cot x$, then $\dfrac{dy}{dx} =$

    (A) $\sec x \csc x$ (B) $\sec x - \csc x$ (C) $\sec x + \csc x$ (D) $\sec^2 x - \csc^2 x$ (E) $\sec^2 x + \csc^2 x$

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9. If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$

    (A) $3x^3 - |x|$ (B) $\left|3x^2 - 1\right|$ (C) $3x^2|x| - 1$ (D) $3|x| - 1$ (E) $3x^2 - 1$

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10. If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

    (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

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11. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

    (A) $9t^2 + 1$

    (B) $3t^2 - 2t + 4$

    (C) $t^3 - t^2 + 4t + 6$

    (D) $t^3 - t^2 + 9t - 20$

    (E) $36t^3 - 4t^2 - 77t + 55$

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12. If f and g are continuous functions, and if $f(x) \geq 0$ for all real numbers x, which of the following must be true?

    I. $\displaystyle\int_a^b f(x)g(x)\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$

    II. $\displaystyle\int_a^b \left(f(x) + g(x)\right)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

    III. $\displaystyle\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)\,dx}$

    (A) I only   (B) II only   (C) III only   (D) II and III only   (E) I, II, and III

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13. The fundamental period of $2\cos(3x)$ is

(A) $\dfrac{2\pi}{3}$ (B) 2π (C) 6π (D) 2   (E) 3

14. $\displaystyle\int \frac{3x^2}{\sqrt{x^3+1}}\,dx =$

(A) $2\sqrt{x^3+1}+C$

(B) $\dfrac{3}{2}\sqrt{x^3+1}+C$

(C) $\sqrt{x^3+1}+C$

(D) $\ln\sqrt{x^3+1}+C$

(E) $\ln(x^3+1)+C$

15. For what value of x does the function $f(x)=(x-2)(x-3)^2$ have a relative maximum?

(A) -3 (B) $-\dfrac{7}{3}$ (C) $-\dfrac{5}{2}$ (D) $\dfrac{7}{3}$ (E) $\dfrac{5}{2}$

16. The slope of the line <u>normal</u> to the graph of $y=2\ln(\sec x)$ at $x=\dfrac{\pi}{4}$ is

(A) -2

(B) $-\dfrac{1}{2}$

(C) $\dfrac{1}{2}$

(D) 2

(E) nonexistent

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17. $\int (x^2 + 1)^2 \, dx =$

    (A) $\dfrac{(x^2 + 1)^3}{3} + C$

    (B) $\dfrac{(x^2 + 1)^3}{6x} + C$

    (C) $\left(\dfrac{x^3}{3} + x\right)^2 + C$

    (D) $\dfrac{2x(x^2 + 1)^3}{3} + C$

    (E) $\dfrac{x^5}{5} + \dfrac{2x^3}{3} + x + C$

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18. If $f(x) = \sin\left(\dfrac{x}{2}\right)$, then there exists a number c in the interval $\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c ?

    (A) $\dfrac{2\pi}{3}$ (B) $\dfrac{3\pi}{4}$ (C) $\dfrac{5\pi}{6}$ (D) π (E) $\dfrac{3\pi}{2}$

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19. Let f be the function defined by $f(x) = \begin{cases} x^3 & \text{for } x \le 0, \\ x & \text{for } x > 0. \end{cases}$ Which of the following statements about f is true?

    (A) f is an odd function.

    (B) f is discontinuous at $x = 0$.

    (C) f has a relative maximum.

    (D) $f'(0) = 0$

    (E) $f'(x) > 0$ for $x \ne 0$

20. Let R be the region in the first quadrant enclosed by the graph of $y = (x+1)^{\frac{1}{3}}$, the line $x = 7$, the x-axis, and the y-axis. The volume of the solid generated when R is revolved about the y-axis is given by

    (A) $\pi \int_0^7 (x+1)^{\frac{2}{3}} dx$

    (B) $2\pi \int_0^7 x(x+1)^{\frac{1}{3}} dx$

    (C) $\pi \int_0^2 (x+1)^{\frac{2}{3}} dx$

    (D) $2\pi \int_0^2 x(x+1)^{\frac{1}{3}} dx$

    (E) $\pi \int_0^7 (y^3 - 1)^2 dy$

21. At what value of x does the graph of $y = \dfrac{1}{x^2} - \dfrac{1}{x^3}$ have a point of inflection?

    (A) 0   (B) 1   (C) 2   (D) 3   (E) At no value of x

22. An antiderivative for $\dfrac{1}{x^2 - 2x + 2}$ is

    (A) $-(x^2 - 2x + 2)^{-2}$

    (B) $\ln(x^2 - 2x + 2)$

    (C) $\ln \left| \dfrac{x-2}{x+1} \right|$

    (D) $\text{arcsec}(x-1)$

    (E) $\arctan(x-1)$

23. How many critical points does the function $f(x) = (x+2)^5 (x-3)^4$ have?

    (A) One   (B) Two   (C) Three   (D) Five   (E) Nine

24. If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then $f'(0)$ is

    (A) $\dfrac{4}{3}$ (B) 0   (C) $-\dfrac{2}{3}$ (D) $-\dfrac{4}{3}$ (E) -2

25. $\dfrac{d}{dx}\left(2^x\right) =$

    (A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\dfrac{2x}{\ln 2}$

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26. A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \dfrac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

    (A) -5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

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27. The function f given by $f(x) = x^3 + 12x - 24$ is

    (A) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
    (B) decreasing for $x < 0$, increasing for $x > 0$
    (C) increasing for all x
    (D) decreasing for all x
    (E) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

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28. $\displaystyle\int_1^{500}\left(13^x - 11^x\right)dx + \int_2^{500}\left(11^x - 13^x\right)dx =$

    (A) 0.000 (B) 14.946 (C) 34.415 (D) 46.000 (E) 136.364

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29. $\displaystyle\lim_{\theta \to 0} \dfrac{1 - \cos\theta}{2\sin^2\theta}$ is

    (A) 0 (B) $\dfrac{1}{8}$ (C) $\dfrac{1}{4}$ (D) 1 (E) nonexistent

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30. The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

    (A) 3π (B) $2\sqrt{3}\,\pi$ (C) $\dfrac{9}{2}\pi$ (D) 9π (E) $\dfrac{36\sqrt{3}}{5}\pi$

31. If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

    (A) $e^{3\ln(x^2)}$ (B) $\dfrac{3}{x^2} e^{3\ln(x^2)}$ (C) $6(\ln x) e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

32. $\displaystyle\int_0^{\sqrt{3}} \dfrac{dx}{\sqrt{4-x^2}} =$

    (A) $\dfrac{\pi}{3}$ (B) $\dfrac{\pi}{4}$ (C) $\dfrac{\pi}{6}$ (D) $\dfrac{1}{2}\ln 2$ (E) $-\ln 2$

33. If $\dfrac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

    (A) $-\dfrac{2}{3}$ (B) $-\dfrac{1}{3}$ (C) 0 (D) $\dfrac{1}{3}$ (E) $\dfrac{2}{3}$

34. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

    (A) $-\dfrac{7}{8}$ feet per minute

    (B) $-\dfrac{7}{24}$ feet per minute

    (C) $\dfrac{7}{24}$ feet per minute

    (D) $\dfrac{7}{8}$ feet per minute

    (E) $\dfrac{21}{25}$ feet per minute

35. If the graph of $y = \dfrac{ax+b}{x+c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then $a + c =$

    (A) -5 (B) -1 (C) 0 (D) 1 (E) 5

36. If the definite integral $\int_0^2 e^{x^2}\,dx$ is first approximated by using two <u>inscribed</u> rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

    (A) 53.60 (B) 30.51 (C) 27.80 (D) 26.80 (E) 12.78

37. If f is a differentiable function, then $f'(a)$ is given by which of the following?

    I. $\displaystyle\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

    II. $\displaystyle\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

    III. $\displaystyle\lim_{x \to a} \frac{f(x+h) - f(x)}{h}$

    (A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

38. If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be $f(x)$?

    (A) $\dfrac{x^3}{3} + \cos x - x + 1$

    (B) $\dfrac{x^3}{3} - \cos x - x + 1$

    (C) $x^3 + \cos x - x + 1$

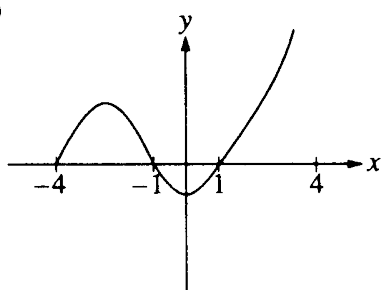   (D) $x^2 - \sin x + 1$

    (E) $x^2 + \sin x + 1$

39. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

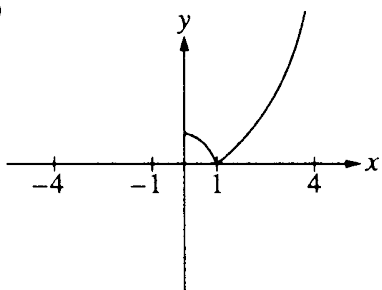   (A) $\dfrac{1}{\pi}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{2}{\pi}$ (D) 1 (E) 2
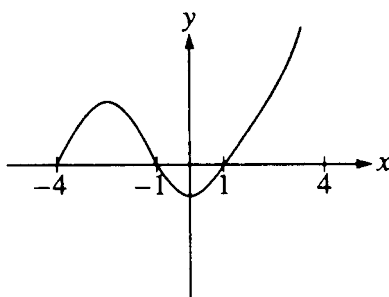
40. The graph of $y = f(x)$ is shown in the figure above. Which of the following could be the graph of $y = f(|x|)$?
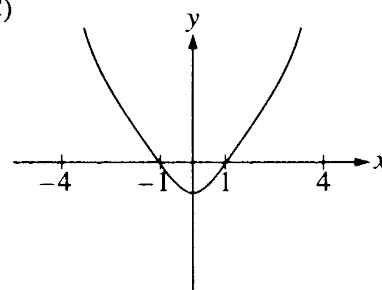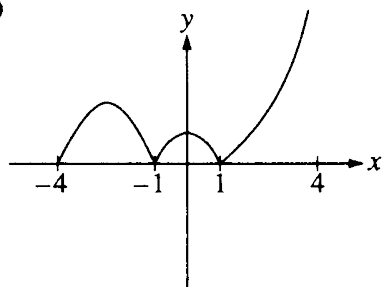
(A)

(B)

(C)

(D)

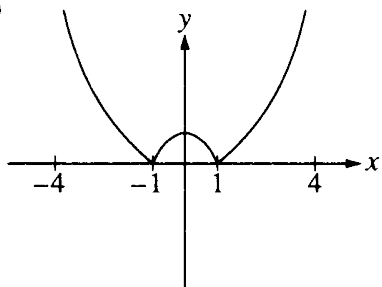(E)

41. $\dfrac{d}{dx} \displaystyle\int_0^x \cos(2\pi u)\,du$ is

(A) 0   (B) $\dfrac{1}{2\pi}\sin x$ (C) $\dfrac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

42. A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

(A) 4.2 pounds   (B) 4.6 pounds   (C) 4.8 pounds   (D) 5.6 pounds   (E) 6.5 pounds

43. $\int x\,f(x)\,dx =$

    (A) $x\,f(x) - \int x\,f'(x)\,dx$

    (B) $\dfrac{x^2}{2}f(x) - \int \dfrac{x^2}{2}f'(x)\,dx$

    (C) $x\,f(x) - \dfrac{x^2}{2}f(x) + C$

    (D) $x\,f(x) - \int f'(x)\,dx$

    (E) $\dfrac{x^2}{2}\int f(x)\,dx$

44. What is the minimum value of $f(x) = x\ln x$?

    (A) $-e$ (B) -1 (C) $-\dfrac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.

45. If Newton's method is used to approximate the real root of $x^3 + x - 1 = 0$, then a first approximation $x_1 = 1$ would lead to a <u>third</u> approximation of $x_3 =$

    (A) 0.682   (B) 0.686   (C) 0.694   (D) 0.750   (E) 1.637