1. C $f'(x) = \frac{3}{2}x^{\frac{1}{2}}$; $f'(4) = \frac{3}{2}\cdot 4^{\frac{1}{2}} = \frac{3}{2}\cdot 2 = 3$

2. B Summing pieces of the form: $(\text{vertical})\cdot(\text{small width})$, vertical $= \left(d - f(x)\right)$, width = Δx

   Area = $\int_a^b \left(d - f(x)\right)dx$

3. D Divide each term by n^3. $\lim\limits_{n\to\infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1} = \lim\limits_{n\to\infty} \frac{3 - \frac{5}{n^2}}{1 - \frac{2}{n} + \frac{1}{n^3}} = 3$

4. A $3x^2 + 3\left(y + x\cdot y'\right) + 6y^2\cdot y' = 0$; $y'(3x + 6y^2) = -(3x^2 + 3y)$

   $y' = -\frac{3x^2 + 3y}{3x + 6y^2} = -\frac{x^2 + y}{x + 2y^2}$

5. A $\lim\limits_{x\to -2} \frac{x^2 - 4}{x + 2} = \lim\limits_{x\to -2} \frac{(x+2)(x-2)}{x+2} = \lim\limits_{x\to -2}(x-2) = -4$. For continuity $f(-2)$ must be –4.

6. D Area = $\int_3^4 \frac{1}{x-1}dx = \left(\ln|x-1|\right)\Big|_3^4 = \ln 3 - \ln 2 = \ln\frac{3}{2}$

7. B $y' = \frac{2\cdot(3x-2) - (2x+3)\cdot 3}{(3x-2)^2}$; $y'(1) = -13$. Tangent line: $y - 5 = -13(x-1) \Rightarrow 13x + y = 18$

8. E $y' = \sec^2 x + \csc^2 x$

9. E $h(x) = f\left(|x|\right) = 3|x|^2 - 1 = 3x^2 - 1$

10. D $f'(x) = 2(x-1)\cdot\sin x + (x-1)^2\cos x$; $f'(0) = (-2)\cdot 0 + 1\cdot 1 = 1$

11. C $a(t) = 6t - 2$; $v(t) = 3t^2 - 2t + C$ and $v(3) = 25 \Rightarrow 25 = 27 - 6 + C$; $v(t) = 3t^2 - 2t + 4$

    $x(t) = t^3 - t^2 + 4t + K$; Since $x(1) = 10$, $K = 6$; $x(t) = t^3 - t^2 + 4t + 6$.

12. B The only one that is true is II. The others can easily been seen as false by examples. For example, let $f(x)=1$ and $g(x)=1$ with $a = 0$ and $b = 2$. Then I gives $2 = 4$ and III gives $2=\sqrt{2}$, both false statements.

13. A period $=\dfrac{2\pi}{B}=\dfrac{2\pi}{3}$

14. A Let $u=x^3+1$. Then $\displaystyle\int\frac{3x^2}{\sqrt{x^3+1}}\,dx=\int u^{-1/2}du=2u^{1/2}+C=2\sqrt{x^3+1}+C$

15. D $f'(x)=(x-3)^2+2(x-2)(x-3)=(x-3)(3x-7)$; $f'(x)$ changes from positive to negative at $x=\dfrac{7}{3}$.

16. B $y'=2\dfrac{\sec x\tan x}{\sec x}=2\tan x$; $y'(\pi/4)=2\tan(\pi/4)=2$. The slope of the normal line $-\dfrac{1}{y'(\pi/4)}=-\dfrac{1}{2}$

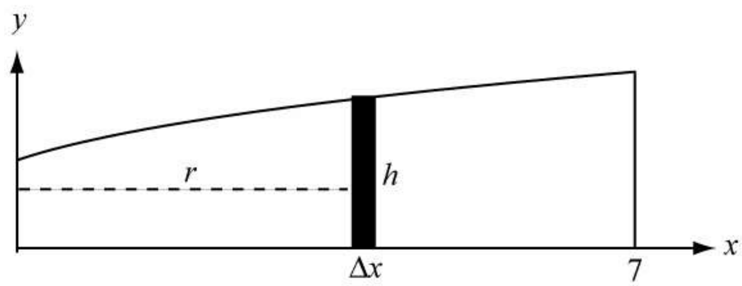
17. E Expand the integrand. $\displaystyle\int(x^2+1)^2\,dx=\int(x^4+2x^2+1)\,dx=\frac{1}{5}x^5+\frac{2}{3}x^3+x+C$

18. D Want c so that $f'(c)=\dfrac{f\left(\frac{3\pi}{2}\right)-f\left(\frac{\pi}{2}\right)}{\frac{3\pi}{2}-\frac{\pi}{2}}=\dfrac{\sin\left(\frac{3\pi}{4}\right)-\sin\left(\frac{\pi}{4}\right)}{\pi}=\dfrac{0}{\pi}$.

$f'(c)=\dfrac{1}{2}\cos\left(\dfrac{c}{2}\right)=0\Rightarrow c=\pi$

19. E The only one that is true is E. A consideration of the graph of $y=f(x)$, which is a standard cubic to the left of 0 and a line with slope 1 to the right of 0, shows the other options to be false.

20. B Use Cylindrical Shells which is no part of the AP Course Description. The volume of each shell is of the form $(2\pi rh)\Delta x$ with $r = x$ and $h = y$. Volume $= 2\pi\int_0^7 x(x+1)^{\frac{1}{3}}\,dx$.



21. C $y = x^{-2} - x^{-3}$; $y' = -2x^{-3} + 3x^{-4}$; $y'' = 6x^{-4} - 12x^{-5} = 6x^{-5}(x-2)$. The only domain value at which there is a sign change in y'' is $x = 2$. Inflection point at $x = 2$.

22. E $\int \frac{1}{x^2 - 2x + 2}\,dx = \int \frac{1}{(x^2 - 2x + 1) + 1}\,dx = \int \frac{1}{(x-1)^2 + 1}\,dx = \tan^{-1}(x-1) + C$

23. C A quick way to do this problem is to use the effect of the multiplicity of the zeros of f on the graph of $y = f(x)$. There is point of inflection and a horizontal tangent at $x = -2$. There is a horizontal tangent and turning point at $x = 3$. There is a horizontal tangent on the interval $(-2,3)$. Thus, there must be 3 critical points. Also, $f'(x) = (x-3)^3(x+2)^4(9x-7)$.

24. A $f'(x) = \frac{2}{3}\left(x^2 - 2x - 1\right)^{-\frac{1}{3}}\left(2x - 2\right)$, $f'(0) = \frac{2}{3}\cdot(-1)\cdot(-2) = \frac{4}{3}$

25. C $\frac{d}{dx}(2^x) = 2^x \cdot \ln 2$

26. D $v(t) = 4\sin t - t$; $a(t) = 4\cos t - 1 = 0$ at $t = \cos^{-1}(1/4) = 1.31812$; $v(1.31812) = 2.55487$

27. C $f'(x) = 3x^2 + 12 > 0$. Thus f is increasing for all x.

28. B $\int_1^{500}(13^x - 11^x)\,dx + \int_2^{500}(11^x - 13^x)\,dx = \int_1^{500}(13^x - 11^x)\,dx - \int_2^{500}(13^x - 11^x)\,dx$

$$= \int_1^2 (13^x - 11^x)\,dx = \left(\frac{13^x}{\ln 13} - \frac{11^x}{\ln 11}\right)\Bigg|_1^2 = \frac{13^2 - 13}{\ln 13} - \frac{11^2 - 11}{\ln 11} = 14.946$$

29. C Use L'Hôpital's Rule (which is no longer part of the AB Course Description).

$$\lim_{\theta\to 0}\frac{1-\cos\theta}{2\sin^2\theta}=\lim_{\theta\to 0}\frac{\sin\theta}{4\sin\theta\cos\theta}=\lim_{\theta\to 0}\frac{1}{4\cos\theta}=\frac{1}{4}$$

A way to do this without L'Hôpital's rule is the following

$$\lim_{\theta\to 0}\frac{1-\cos\theta}{2\sin^2\theta}=\lim_{\theta\to 0}\frac{1-\cos\theta}{2(1-\cos^2\theta)}=\lim_{\theta\to 0}\frac{1-\cos\theta}{2(1-\cos\theta)(1+\cos\theta)}=\lim_{\theta\to 0}\frac{1}{2(1+\cos\theta)}=\frac{1}{4}$$

30. C Each slice is a disk whose volume is given by $\pi r^2\Delta x$, where $r=\sqrt{x}$.

$$\text{Volume}=\pi\int_0^3(\sqrt{x})^2\,dx=\pi\int_0^3 x\,dx=\frac{\pi}{2}x^2\Big|_0^3=\frac{9}{2}\pi.$$

31. E $f(x)=e^{3\ln(x^2)}=e^{\ln(x^6)}=x^6;\ f'(x)=6x^5$

32. A $\displaystyle\int\frac{du}{\sqrt{a^2-u^2}}=\sin^{-1}\left(\frac{u}{a}\right)+C,\ a>0$

$$\int_0^{\sqrt{3}}\frac{dx}{\sqrt{4-x^2}}=\sin^{-1}\left(\frac{x}{2}\right)\Big|_0^{\sqrt{3}}=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)-\sin^{-1}(0)=\frac{\pi}{3}$$

33. B Separate the variables. $y^{-2}dy=2dx$; $-\frac{1}{y}=2x+C$; $y=\frac{-1}{2x+C}$. Substitute the point $(1,-1)$ to find the value of C. Then $-1=\frac{-1}{2+C}\Rightarrow C=-1$, so $y=\frac{1}{1-2x}$. When $x=2$, $y=-\frac{1}{3}$.

34. D Let x and y represent the horizontal and vertical sides of the triangle formed by the ladder, the wall, and the ground.

$$x^2+y^2=25;\ 2x\frac{dx}{dt}+2y\frac{dy}{dt}=0;\ 2(24)\frac{dx}{dt}+2(7)(-3)=0;\ \frac{dx}{dt}=\frac{7}{8}.$$

35. E For there to be a vertical asymptote at $x=-3$, the value of c must be 3. For $y=2$ to be a horizontal asymptote, the value of a must be 2. Thus $a+c=5$.

36. D Rectangle approximation = $e^0+e^1=1+e$

Trapezoid approximation. = $\left(1+2e+e^4\right)/2$.

Difference = $(e^4-1)/2=26.799$.

37. C I and II both give the derivative at a. In III the denominator is fixed. This is not the derivative of f at $x = a$. This gives the slope of the secant line from $\left(a, f(a)\right)$ to $\left(a+h, f(a+h)\right)$.

38. A $f'(x) = x^2 - \sin x + C$, $f(x) = \frac{1}{3}x^3 + \cos x + Cx + K$. Option A is the only one with this form.

39. D $A = \pi r^2$ and $C = 2\pi r$; $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ and $\frac{dC}{dt} = 2\pi \frac{dr}{dt}$. For $\frac{dA}{dt} = \frac{dC}{dt}$, $r = 1$.

40. C The graph of $y = f\left(|x|\right)$ is symmetric to the y-axis. This leaves only options C and E. For $x > 0$, x and $|x|$ are the same, so the graphs of $f(x)$ and $f\left(|x|\right)$ must be the same. This is option C.

41. D Answer follows from the Fundamental Theorem of Calculus.

42. B This is an example of exponential growth. We know from pre-calculus that $w = 2\left(\frac{3.5}{2}\right)^{\frac{t}{2}}$ is an exponential function that meets the two given conditions. When $t = 3$, $w = 4.630$. Using calculus the student may translate the statement "increasing at a rate proportional to its weight" to mean exponential growth and write the equation $w = 2e^{kt}$. Using the given conditions, $3.5 = 2e^{2k}$; $\ln(1.75) = 2k$; $k = \frac{\ln(1.75)}{2}$; $w = 2e^{t \cdot \frac{\ln(1.75)}{2}}$. When $t = 3$, $w = 4.630$.

43. B Use the technique of antiderivative by parts, which is no longer in the AB Course Description. The formula is $\int u\,dv = uv - \int v\,du$. Let $u = f(x)$ and $dv = x\,dx$. This leads to $\int x f(x)\,dx = \frac{1}{2}x^2 f(x) - \frac{1}{2}\int x^2 f'(x)\,dx$.

44. C $f'(x) = \ln x + x \cdot \frac{1}{x}$; $f'(x)$ changes sign from negative to positive only at $x = e^{-1}$. $f(e^{-1}) = -e^{-1} = -\frac{1}{e}$.

45. B Let $f(x) = x^3 + x - 1$. Then Newton's method (which is no longer part of the AP Course Description) gives

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^{\,3} + x_n - 1}{3x_n^{\,2} + 1}$$

$$x_2 = 1 - \frac{1+1-1}{3+1} = \frac{3}{4}$$

$$x_3 = \frac{3}{4} - \frac{\left(\frac{3}{4}\right)^3 + \frac{3}{4} - 1}{3\left(\frac{3}{4}\right)^2 + 1} = \frac{59}{86} = 0.686$$