1997 Calculus AB Solutions: Part A

1. C $\quad \int_1^2 (4x^3 - 6x)\,dx = (x^4 - 3x^2)\Big|_1^2 = (16-12) - (1-3) = 6$

2. A $\quad f(x) = x(2x-3)^{\frac{1}{2}}; \quad f'(x) = (2x-3)^{\frac{1}{2}} + x(2x-3)^{-\frac{1}{2}} = (2x-3)^{-\frac{1}{2}}(3x-3) = \dfrac{(3x-3)}{\sqrt{2x-3}}$

3. C $\quad \int_a^b (f(x)+5)\,dx = \int_a^b f(x)dx + 5\int_a^b 1\,dx = a + 2b + 5(b-a) = 7b - 4a$

4. D $\quad f(x) = -x^3 + x + \dfrac{1}{x}; \quad f'(x) = -3x^2 + 1 - \dfrac{1}{x^2}; \quad f'(-1) = -3(-1)^2 + 1 - \dfrac{1}{(-1)^2} = -3 + 1 - 1 = -3$

5. E $\quad y = 3x^4 - 16x^3 + 24x^2 + 48; \quad y' = 12x^3 - 48x^2 + 48x; \quad y'' = 36x^2 - 96x + 48 = 12(3x-2)(x-2)$

   $y'' < 0$ for $\dfrac{2}{3} < x < 2,$ therefore the graph is concave down for $\dfrac{2}{3} < x < 2$

6. C $\quad \dfrac{1}{2}\int e^{\frac{t}{2}}\,dt = e^{\frac{t}{2}} + C$

7. D $\quad \dfrac{d}{dx}\cos^2(x^3) = 2\cos(x^3)\left(\dfrac{d}{dx}(\cos(x^3)\right) = 2\cos(x^3)(-\sin(x^3)\left(\dfrac{d}{dx}(x^3)\right)$

   $= 2\cos(x^3)(-\sin(x^3)(3x^2)$

8. C The bug change direction when v changes sign. This happens at $t = 6$.

9. B Let A_1 be the area between the graph and t-axis for $0 \le t \le 6$, and let A_2 be the area between the graph and the t-axis for $6 \le t \le 8$ Then $A_1 = 12$ and $A_2 = 1$. The total distance is $A_1 + A_2 = 13$.

10. E $\quad y = \cos(2x); \quad y' = -2\sin(2x); \quad y'\left(\dfrac{\pi}{4}\right) = -2$ and $y\left(\dfrac{\pi}{4}\right) = 0; \quad y = -2\left(x - \dfrac{\pi}{4}\right)$

11. E Since f' is positive for $-2 < x < 2$ and negative for $x < -2$ and for $x > 2,$ we are looking for a graph that is increasing for $-2 < x < 2$ and decreasing otherwise. Only option E.

12. B $\quad y = \dfrac{1}{2}x^2; \quad y' = x;$ We want $y' = \dfrac{1}{2} \Rightarrow x = \dfrac{1}{2}$. So the point is $\left(\dfrac{1}{2}, \dfrac{1}{8}\right)$.

13. A $f'(x) = \dfrac{\left|4 - x^2\right|}{x - 2}$; f is decreasing when $f' < 0$. Since the numerator is non-negative, this is only when the denominator is negative. Only when $x < 2$.

14. C $f(x) \approx L(x) = 2 + 5(x - 3)$; $L(x) = 0$ if $0 = 5x - 13 \Rightarrow x = 2.6$

15. B Statement B is true because $\lim\limits_{x \to a^-} f(x) = 2 = \lim\limits_{x \to a^+} f(x)$. Also, $\lim\limits_{x \to b} f(x)$ does not exist because the left- and right-sided limits are not equal, so neither (A), (C), nor (D) are true.

16. D The area of the region is given by $\int_{-2}^{2} (5 - (x^2 + 1))dx = 2(4x - \frac{1}{3}x^3)\Big|_0^2 = 2\left(8 - \frac{8}{3}\right) = \frac{32}{3}$

17. A $x^2 + y^2 = 25$; $2x + 2y \cdot y' = 0$; $x + y \cdot y' = 0$; $y'(4,3) = -\frac{4}{3}$;

$x + y \cdot y' = 0 \Rightarrow 1 + y \cdot y'' + y' \cdot y' = 0$; $1 + (3)y'' + \left(-\frac{4}{3}\right) \cdot \left(-\frac{4}{3}\right) = 0$; $y'' = -\frac{25}{27}$

18. C $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} dx$ is of the form $\int e^u du$ where $u = \tan x$..

$\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} dx = e^{\tan x}\Big|_0^{\frac{\pi}{4}} = e^1 - e^0 = e - 1$

19. D $f(x) = \ln\left|x^2 - 1\right|$; $f'(x) = \frac{1}{x^2 - 1} \cdot \frac{d}{dx}(x^2 - 1) = \frac{2x}{x^2 - 1}$

20. E $\frac{1}{8}\int_{-3}^{5} \cos x\, dx = \frac{1}{8}(\sin 5 - \sin(-3)) = \frac{1}{8}(\sin 5 + \sin 3)$; Note: Since the sine is an odd function, $\sin(-3) = -\sin(3)$.

21. E $\lim\limits_{x \to 1} \frac{x}{\ln x}$ is nonexistent since $\lim\limits_{x \to 1} \ln x = 0$ and $\lim\limits_{x \to 1} x \neq 0$.

22. D $f(x) = (x^2 - 3)e^{-x}$; $f'(x) = e^{-x}(-x^2 + 2x + 3) = -e^{-x}(x - 3)(x + 1)$; $f'(x) > 0$ for $-1 < x < 3$

23. A Disks where $r = x$. $V = \pi\int_0^2 x^2 dy = \pi\int_0^2 y^4 dy = \frac{\pi}{5}y^5\Big|_0^2 = \frac{32\pi}{5}$

24. B Let $[0,1]$ be divided into 50 subintervals. $\Delta x = \frac{1}{50}$; $x_1 = \frac{1}{50}, x_2 = \frac{2}{50}, x_3 = \frac{3}{50}, \cdots, x_{50} = 1$

Using $f(x) = \sqrt{x}$, the right Riemann sum $\sum_{i=1}^{50} f(x_i)\Delta x$ is an approximation for $\int_0^1 \sqrt{x}\, dx$.

25. A Use the technique of antiderivatives by parts, which was removed from the AB Course Description in 1998.

$$u = x \qquad dv = \sin 2x\, dx$$
$$du = dx \qquad v = -\frac{1}{2}\cos 2x$$

$$\int x\sin(2x)\, dx = -\frac{1}{2}x\cos(2x) + \int \frac{1}{2}\cos(2x)\, dx = -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C$$

76. E $f(x)=\frac{e^{2x}}{2x}$; $f'(x)=\frac{2e^{2x}\cdot 2x-2e^{2x}}{4x^2}=\frac{e^{2x}(2x-1)}{2x^2}$

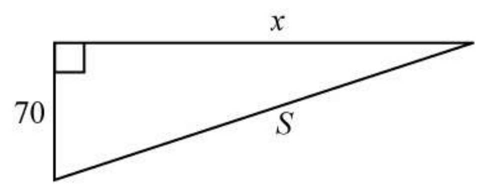
77. D $y=x^3+6x^2+7x-2\cos x$. Look at the graph of $y''=6x+12+2\cos x$ in the window [–3,–1] since that domain contains all the option values. y'' changes sign at $x=-1.89$.

78. D $F(3)-F(0)=\int_0^3 f(x)dx=\int_0^1 f(x)dx+\int_1^3 f(x)dx=2+2.3=4.3$
(Count squares for $\int_0^1 f(x)dx$)

79. C The stem of the questions means $f'(2)=5$. Thus f is differentiable at $x = 2$ and therefore continuous at $x = 2$. We know nothing of the continuity of f'. I and II only.

80. A $f(x)=2e^{4x^2}$; $f'(x)=16xe^{4x^2}$; We want $16xe^{4x^2}=3$. Graph the derivative function and the function $y = 3$, then find the intersection to get $x=0.168$.

81. A Let x be the distance of the train from the crossing. Then $\frac{dx}{dt}=60$.



$S^2=x^2+70^2\Rightarrow 2S\frac{dS}{dt}=2x\frac{dx}{dt}\Rightarrow\frac{dS}{dt}=\frac{x}{S}\frac{dx}{dt}$.
After 4 seconds, $x=240$ and so $S=250$.
Therefore $\frac{dS}{dt}=\frac{240}{250}(60)=57.6$

82. B $P(x)=2x^2-8x$; $P'(x)=4x-8$; P' changes from negative to positive at $x = 2$. $P(2)=-8$

83. C $\cos x=x$ at $x=0.739085$. Store this in A. $\int_0^A(\cos x-x)\,dx=0.400$

84. C Cross sections are squares with sides of length y.
Volume $=\int_1^e y^2dx=\int_1^e \ln x\,dx=(x\ln x-x)\Big|_1^e=(e\ln e-e)-(0-1)=1$

85. C Look at the graph of f' and locate where the y changes from positive to negative. $x=0.91$

86. A $f(x)=\sqrt{x}$; $f'(x)=\frac{1}{2\sqrt{x}}$; $\frac{1}{2\sqrt{c}}=2\cdot\frac{1}{2\sqrt{1}}\Rightarrow c=\frac{1}{4}$

87. B $a(t) = t + \sin t$ and $v(0) = -2 \Rightarrow v(t) = \frac{1}{2}t^2 - \cos t - 1$; $v(t) = 0$ at $t = 1.48$

88. E $f(x) = \int_a^x h(x)dx \Rightarrow f(a) = 0,$ therefore only (A) or (E) are possible. But $f'(x) = h(x)$ and therefore f is differentiable at $x = b$. This is true for the graph in option (E) but not in option (A) where there appears to be a corner in the graph at $x = b$. Also, Since h is increasing at first, the graph of f must start out concave up. This is also true in (E) but not (A).

89. B $T = \frac{1}{2} \cdot \frac{1}{2}(3 + 2 \cdot 3 + 2 \cdot 5 + 2 \cdot 8 + 13) = 12$

90. D

$F(x) = \frac{1}{2}\sin^2 x$	$F'(x) = \sin x \cos x$	Yes
$F(x) = \frac{1}{2}\cos^2 x$	$F'(x) = -\cos x \sin x$	No
$F(x) = -\frac{1}{4}\cos(2x)$	$F'(x) = \frac{1}{2}\sin(2x) = \sin x \cos x$	Yes