A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b.

(b) If $\int_0^1 f(x)\,dx = 32$, what is the value of k?

$f'(-1) = 0$

$f''(-2) = 0$

(a)

$f'(x) = 12x^2 + 2ax + b$ $\longrightarrow$ $f'(x) = 12x^2 + 2(24)x + b$

$f''(x) = 24x + 2a$ $\qquad\qquad f'(x) = 12x^2 + 48x + b$

$\boxed{f''(-2) = 0}$ $\qquad\qquad \boxed{f'(-1) = 0}$

$0 = 24(-2) + 2a$ $\qquad\qquad 0 = 12(-1)^2 + 48(-1) + b$

$0 = -48 + 2a$ $\qquad\qquad 0 = 12(1) - 48 + b$

$48 = 2a$ $\qquad\qquad 0 = -36 + b$

$a = 24$ $\qquad\qquad 36 = b$

$a = 24$
$b = 36$

(b) $\qquad\qquad \int_0^1 f(x)\,dx = 32$

$\int_0^1 (4x^3 + 24x^2 + 36x + k)\,dx = 32$

$\left(x^4 + 8x^3 + 18x^2 + kx\right)\Big|_0^1 = 32$

$\left[(1)^4 + 8(1)^3 + 18(1)^2 + k(1)\right] - \left[(0)^4 + 8(0)^3 + 18(0)^2 + k(0)\right] = 32$

$\left[1 + 8 + 18 + k\right] - \left[0\right] = 32$

$27 + k \qquad\qquad = 32$

$k = 5$