2002 AP® CALCULUS AB
Question 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(a) $\int_0^{1.5} (3f'(x)+4)dx = [3\cdot f(x) + 4x]\Big|_0^{1.5}$

$= [3f(1.5) + 4(1.5)] - [3\cdot f(0) + 4(0)]$

$= [3(-1) + 6] - [3\cdot(-7) + 0]$

$= 3 + +21$

$= 24$

(b)

Point	Slope	Tangent line
$(1,-4)$	$m=5$	$y+4=5(x-1)$

$f(1.2) \approx -3$

$$\boxed{\begin{aligned} y+4 &= 5(1.2-1) \\ y &= 5(0.2)-4 \\ y &= 1-4 \\ y &= -3 \end{aligned}}$$

This approximation is less than $f(1.2)$ because the graph of $f(x)$ is concave up on $(-1.5, 1.5)$.

(c) The Mean Value Theorem guarantees a value of c on $(0, 0.5)$ such that $f''(c) = r$

$$r = f''(c) = \frac{f'(0.5) - f'(0)}{0.5 - 0} = \frac{3-0}{0.5-0} = \frac{3}{0.5} = 6$$