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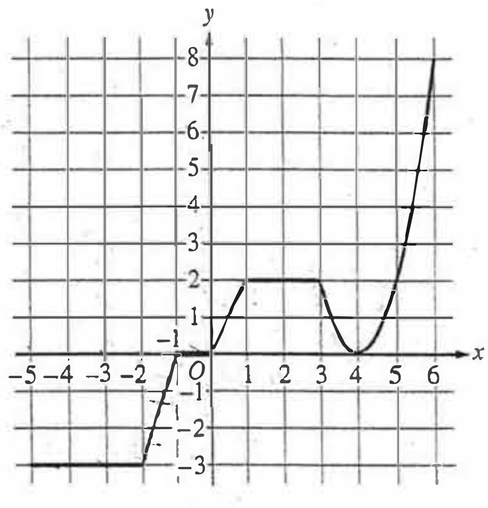

Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$f'(x) = g(x)$

$f(x) = \int_1^x g(t)\,dt + 3$

$f(-5) = -\int_{-5}^1 g(t)\,dt + 3$

$f(-5) = -\left(\frac{1}{2}(1)(2) - \frac{1}{2}(1)(3) - (3)(3)\right) + 3$

$f(-5) = -\left(1 - \frac{3}{2} - 9\right) + 3$

$f(-5) = 8 + \frac{3}{2} + 3$

$f(-5) = 11 + \frac{3}{2}$

$18/3 = 6$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$\int_1^6 g(x)dx = \int_1^3 g(x)dx + \int_3^6 g(x)dx$

$\int_1^6 g(x)dx = (2)(2) + \int_3^6 2(x-4)^2 dx$

$\int_1^6 g(x)dx = 4 + \left(\frac{2}{3}(x-4)^3\right)\Big|_3^6$

$\int_1^6 g(x)dx = 4 + \left(\frac{2}{3}(8) - \frac{2}{3}(-1)\right)$

$\int_1^6 g(x)dx = 4 + \frac{16}{3} + \frac{2}{3}$

$\int_1^6 g(x)dx = 10$

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(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

On the interval $(0,1) \cup (4,6)$ f is both increasing and concave up since $f'(x) = g(x)$ and g is positive on the that interval meaning f is increasing on that interval, and g is increasing on that interval, meaning $f''(x) > 0$ on that interval, therefore f is concave up on that interval

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

f has a point of inflection at $x=4$ since $f'(x) = g(x)$ and since g switches from decreasing to increasing at $x=4$, therefore $f''(4) = 0$ at that point and would change signs from $\ominus$ to $\oplus$ at $x=4$, therefore $x=4$ is an inflection point.

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