2021

AB1 / BC1

These point distributions are pure speculation.

(a) $f'(2.25) \approx \frac{10-6}{2.5-2}$
2 points
$= 8$

2.25 cm from the center of the petri dish this is the rate at which the density of bacteria is increasing in mg/cm² per cm of distance from the center.

(b) $2\pi(1 \cdot 1 \cdot 2 + 1 \cdot 2 \cdot 6 + .5 \cdot 2.5 \cdot 10 + 1.5 \cdot 4 \cdot 18) = 269\pi$
3 points
$= 845.088$

(c) It is an overestimate because it is a right Riemann Sum and $r f(r)$ is increasing.
1 point

(d) $\frac{1}{3}\int_1^4 g(r)\,dr = g(k)$
3 points
$9.876 = g(k)$
$k = 2.497$

AB 2

(a) position of $P = 5 + \int_0^1 V_p(t)\,dt$
3 points
$= 5.370$ or 5.371

position of $Q = 10 + \int_0^1 V_q(t)\,dt$
$= 8.564$

(b) $V_p(1) = .841 > 0$
2 points
$V_q(1) = -1 < 0$

They are moving toward each other because P is moving right and Q starts right of P and moves left.

(c) $a_q(1) = V_q'(1)$
2 points
$= 1.026$ or 1.027

The speed of Q is decreasing because $V_q(1)$ and $a_q(1)$ have opposite signs.

(d) $\text{tot. dist}_p = \int_0^\pi |V_p(t)|\,dt$
2 points
$= 1.931$

AB3 / BC3

(a) $6x\sqrt{4-x^2}=0$

3points

$$x=0,\pm 2$$

$$\text{area} = \int_0^2 6x\,(4-x^2)^{\frac{1}{2}}dx$$

$$= -3\int_0^2 -2x\,(4-x^2)^{\frac{1}{2}}dx$$

$$= -3\cdot\tfrac{2}{3}\,(4-x^2)^{3/2}\Big|_0^2$$

$$= -2\cdot 0 + 2\cdot 4^{3/2}$$

$$= 16$$

(b) $\dfrac{dy}{dx} = \dfrac{c(4-2x^2)}{\sqrt{4-x^2}} = 0$

2points

$$x=\sqrt{2}$$

$$y = c\sqrt{2}\,\sqrt{4-(\sqrt{2})^2}$$

$$=2c$$

$$2c = 1.2$$

$$c = .6$$

(c) $\text{Vol.} = \pi\int_0^2 \left(cx\sqrt{4-x^2}\right)^2 dx$

4points

$$= \pi\int_0^2 c^2x^2(4-x^2)\,dx$$

$$= c^2\pi\int_0^2 (4x^2-x^4)\,dx$$

$$= c^2\pi\left(\tfrac{4}{3}x^3-\tfrac{1}{5}x^5\right)\Big|_0^2$$

$$= c^2\pi\left(\tfrac{32}{3}-\tfrac{32}{5}\right)$$

$$c^2\pi\left(\tfrac{32}{3}-\tfrac{32}{5}\right) = 2\pi$$

$$c = \sqrt{\frac{2}{\frac{32}{3}-\frac{32}{5}}}$$

$$= \sqrt{\frac{15}{32}}$$

AB4 / BC4

(a) $G'(x) = f(x)$

2 points

$G''(x) = f'(x)$

G is concave up on $(-4,-2)$ and $(2,6)$ because f is increasing.

(b) $p'(x) = G(x)\, f'(x) + f(x)\, G'(x)$

3 points

$p'(3) = \int_0^3 f(t)\,dt \cdot 1 + (-3)(-3)$

$= -\frac{1}{2} \cdot 1 (3+4) + 9$

$= \frac{11}{2}$

(c) $\lim_{x \to 2} G(x) = \int_0^2 f(t)\,dt$

2 points

$= 0$

$\lim_{x \to 2} (x^2 - 2x) = 0$

$\lim_{x \to 2} \frac{G(x)}{x^2 - 2x} = \lim_{x \to 2} \frac{f(x)}{2x - 2}$

$= \frac{f(2)}{2}$

$= -2$

(d) $G(-4) = \int_0^{-4} f(t)\,dt$

3 points

$= -\left(\frac{1}{2} \cdot 2 \cdot 6 + \frac{1}{2} \cdot 2 (6+4)\right)$

$= -16$

$G(2) = \int_0^2 f(t)\,dt$

$= 0$

$AROC = \frac{0 + 16}{2 + 4}$

$= \frac{8}{3}$

MVT does guarantee a c-value because G is both continuous and differentiable on $[-4, 2]$.

AB 5

(a) $4y \frac{dy}{dx} = y\cos x + \sin x \frac{dy}{dx}$

2 points

$\frac{dy}{dx}(4y - \sin x) = y\cos x$

$\frac{dy}{dx} = \frac{y\cos x}{4y - \sin x}$

(b) $\left.\frac{dy}{dx}\right|_{(0,\sqrt{3})} = \frac{\sqrt{3}\cos 0}{4\sqrt{3} - \sin 0}$

2 points

$= \frac{1}{4}$

Tan. Line $\quad y - \sqrt{3} = \frac{1}{4}(x - 0)$

(c) $y\cos x = 0$

2 points

$x = \frac{\pi}{2}$

$2y^2 - 6 = y\sin\frac{\pi}{2}$

$2y^2 - y - 6 = 0$

$(2y+3)(y-2) = 0$ $\quad$ The point is $(\frac{\pi}{2}, 2)$.

$\cancel{y = -\frac{3}{2}} \quad y = 2$

(d) $\frac{d^2y}{dx^2} = \frac{(4y - \sin x)(y\sin x) + \cos x \cdot \frac{dy}{dx}}{(4y - \sin x)^2}$

3 points

$\left.\frac{d^2y}{dx^2}\right|_{(\frac{\pi}{2},2)} = \frac{2\sin\frac{\pi}{2}}{4\cdot 2 - \sin\frac{\pi}{2}}$ $\quad$ since $\frac{dy}{dx} = 0$

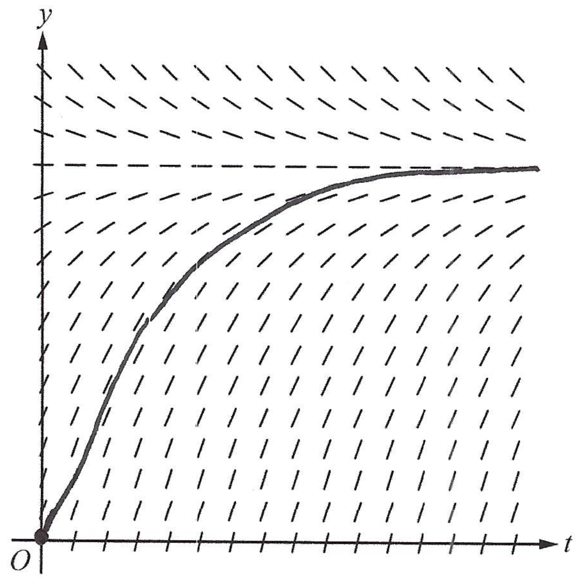
$= \frac{2}{7} > 0$

f has a rel. min. at $(\frac{\pi}{2}, 2)$.

AB 6

(a)

1 point



(b) The amount of medication in the patient is approaching 12 mg.

1 point

(c) $\frac{1}{12-y}\,dy = \frac{1}{3}\,dt$

4 points

$-\ln(12-y) = \frac{1}{3}t + C$

$\ln(12-y = -\frac{1}{3}t + C$

$12-y = C\,e^{-\frac{1}{3}t}$

$(0,0) \rightarrow \quad 12 = C$

$y = -12\,e^{-\frac{1}{3}t} + 12$

(d) $\frac{d^2y}{dt^2} = 0 - \frac{(t+2)\frac{dy}{dt} - y}{(t+2)^2}$

3 points

$\left.\frac{d^2y}{dt^2}\right|_{\substack{t=1\\y=2.5}} = -\frac{3\left(3-\frac{2.5}{3}\right) - 2.5}{9}$

$= -\frac{\frac{13}{2} - \frac{5}{2}}{9}$

$= -\frac{4}{9} < 0$

The rate of change of the amt. of medication is decreasing.

BC 2

(a) $\text{speed}\Big|_{t=1.2} = \sqrt{(x'(1.2))^2 + (y'(1.2))^2}$ $\qquad x'(t) = (t-1)e^{t^2}$

3 points $\qquad y'(t) = \sin t^{1.25}$

$= 1.271$

$\text{acc.} = \langle x''(t), y''(t) \rangle$

$\text{acc.}\Big|_{t=1.2} = \langle 6.246 \text{ or } 6.247, .405 \rangle$

(b) $\text{Tot. dist.} = \int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$

2 points

$= 1.009 \text{ or } 1.010$

(c) $(t-1)e^{t^2} = 0$

4 points $\quad t = 1$

$x'(t) < 0$ on $[0, 1)$ and $x'(t) > 0$ on $(1, \infty)$

The particle is farthest left at $t=1$ but continues right for all times after $t=1$,

$x(1) = -2.603 \text{ or } -2.604$

$y(1) = 5.410$

BC 5

(a) $f'(1) = 4\,(1 \ln 1)$
$= 0$

2pts

$f(x) \approx 4 + \frac{4}{2!}(x-1)^2$

$f(2) \approx 4 + 2$
$= 6$

(b)

2pts

x	y	$m = y\,(x \ln x)$	$\Delta y = m\,\Delta x$
1	4	$m = 0$	$\Delta y = 0$
1.5	4	$m = 4\,(1.5 \ln 1.5)$	$\Delta y = 4\,(1.5 \ln 1.5)\cdot .5$ $= 3 \ln 1.5$
2			

$f(2) \approx 4 + 3 \ln 1.5$

(c)

5pts

$\frac{1}{y}\,dy = x \ln x\, dx$ $\quad u = \ln x \quad dv = x\,dx$
$\quad du = \frac{1}{x}dx \quad v = \frac{1}{2}x^2$

$\ln y = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx$

$\ln y = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

$y = C\,e^{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2}$

$(1, 4) \rightarrow 4 = C\,e^{-\frac{1}{4}}$

$4e^{\frac{1}{4}} = C$

$y = 4e^{\frac{1}{4}}\,e^{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2}$

BC 6

(a) $f(x) = e^{-x}$ is positive, continuous, and decreasing

3pts

$$\lim_{b\to\infty} \int_0^b e^{-x}dx = \lim_{b\to\infty} -e^{-x}\Big|_0^b$$

$$= \lim_{b\to\infty} \frac{-1}{e^b} + 1$$

$$= 1$$

The series also converges.

(b) $\lim_{n\to\infty} \left(\frac{1}{e^n} \cdot \frac{2e^n+3}{1} \right) = 2$

2pts

$\sum_{n=0}^{\infty} \frac{1}{2e^n+3}$ also converges

$\sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n+3}$ converges absolutely

(c) $\lim_{n\to\infty} \left| \frac{x^{n+1}}{2e^{n+1}+3} \cdot \frac{2e^n+3}{x^n} \right| = \left| \frac{x}{e} \right| < 1$

3 pts

$|x| < e$

the radius of convergence is e.

(d) Alt. Series Error $\leq \frac{1}{2e^2+3}$

1pt