The Lovely Ladybug

Name:_____

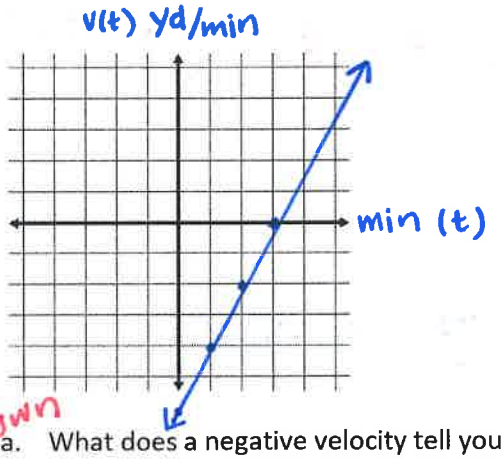
A ladybug moves along a branch so that its position from the trunk of the tree at time t is given by $s(t) = t^2 - 6t + 5$, for $0 \le t \le 5$, where $s(t)$ is measured in yards and t is measured in minutes.

1. Find an equation for the velocity of the ladybug, $v(t)$.

$v(t) = s'(t)$

$v(t) = s'(t) = 2t - 6$

2. Sketch the graph of $v(t)$. Use proper units to label your axes.



a. What does a negative velocity tell you about the direction the ladybug is walking?

$v(t) < 0$ object is moving left/down

ladybug is going backwards (towards trunk)

b. What does a positive velocity tell you about the direction the ladybug is walking?

$v(t) > 0$ object is moving right/up

ladybug is going forward (away from trunk)

3. At what time is the ladybug at rest? How do you know?

$v(t) = 0$ object is at rest

At $t = 3$ minutes because the velocity is 0.

4. Let's look at what is happening with the ladybug at $t = 1$.

a. Find $v(1)$. Use proper units.

$v(1) = 2(1) - 6 = -4$ yd/min

b. Find $v'(1)$. What does your answer mean in the context of this problem? Use proper units.

$v'(t) = a(t)$

$v'(t) = 2 \quad v'(1) = 2 \ \frac{\text{yd/min}}{\text{min}}$ acceleration!

At $t = 1$, the velocity is increasing at a rate of 2 yd/min per minute.

c. At $t = 1$, is the ladybug speeding up or slowing down? How do you know?

If $v(t)$ and $a(t)$ have opposite signs, object is slowing down.

The ladybug is slowing down. The velocity and acceleration are working in opposite directions.

5. Off on another branch of the tree, a worm is crawling at a velocity given by $v(t) = -4t + 3$ where $v(t)$ is measured in yards per minute, and t is in minutes. Find a possible function that would give the position, $s(t)$, of the worm.

antiderivative ↓ $v(t) = -4t + 3$ ↑ derivative

$s(t) = -2t^2 + 3t + C$

CALC MEDIC

Topic 4.2—Position, Velocity, and Acceleration

Important Ideas:

(1) Position
$s(t)$

Velocity = speed w/ direction
$v(t) = s'(t)$

Acceleration
$a(t) = v'(t) = s''(t)$

(2) If $v(t) = 0$, object is at rest
If $v(t) > 0$, object is moving right or up
If $v(t) < 0$, object is moving left or up

(3)

	$a(t) > 0$	$a(t) < 0$
$v(t) > 0$	speeding up	slowing down
$v(t) < 0$	slowing down	speeding up

Check Your Understanding!

1. The position of a yo-yo is given by $H(t) = t^3 - 6t^2 + 5t + 30$, where t is measured in seconds and $H(t)$ is measured in inches.
   a. Find the average velocity of the yo-yo over the first four seconds.

$$\frac{H(4) - H(0)}{4 - 0} = \frac{18 - 30}{4} = \frac{-12}{4} = -3 \text{ in/sec}$$

   b. Find the instantaneous velocity of the yo-yo at $t = 3$ seconds.

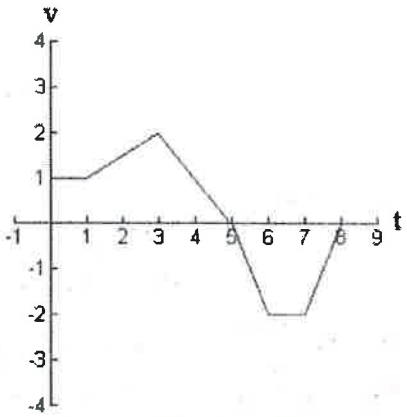
$$H'(t) = 3t^2 - 12t + 5$$
$$H'(3) = 3(3)^2 - 12(3) + 5 = 27 - 36 + 5 = -4 \text{ in/sec}$$

2. The position of an object is given by $x(t) = \cos(3t) - \sin(4t)$. Find the acceleration at $t = 0$.

$$v(t) = x'(t) = -3\sin(3t) - 4\cos(4t)$$
$$a(t) = v'(t) = -9\cos(3t) + 4\sin(4t) \qquad a(0) = -9\cos(0) + 4\sin(0) = \boxed{-9}$$

3. The graph of $v(t)$ is shown below, representing the velocity of an object moving on a line over the time interval [0,8].
   a. When is the object at rest? Justify your answer.

The object is at rest at $t = 5$ and $t = 8$ because the velocity, $v(t)$, is equal to 0.



   b. At $t = 2$, is the object speeding up or slowing down? Explain your answer.

$v(2) > 0$
$a(2) = v'(2) > 0$ since the slope of $v(t)$ is positive at $t = 2$.
Since the velocity and acceleration have the same sign, the object is speeding up.

4. Let $v(t) = \frac{1}{\pi} + \sin(3t)$ represent the velocity of an object moving on a line. On the interval $[\frac{\pi}{2}, \pi]$, what is the velocity when the acceleration is 3?

$$a(t) = v'(t) = 3\cos(3t)$$
$$3 = 3\cos(3t) \Rightarrow \cos(3t) = 1 \Rightarrow 3t = 0, 2\pi, 4\pi \ldots \Rightarrow t = \frac{2\pi}{3}$$
$$v\left(\frac{2\pi}{3}\right) = \frac{1}{\pi} + \sin(2\pi) = \boxed{\frac{1}{\pi}}$$