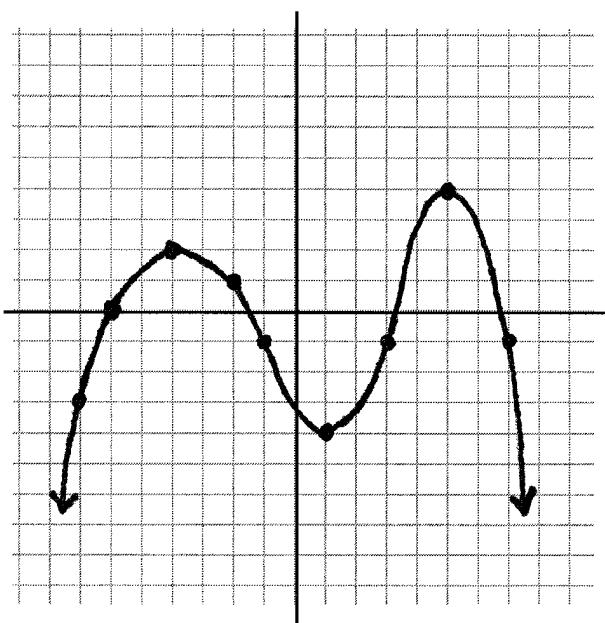
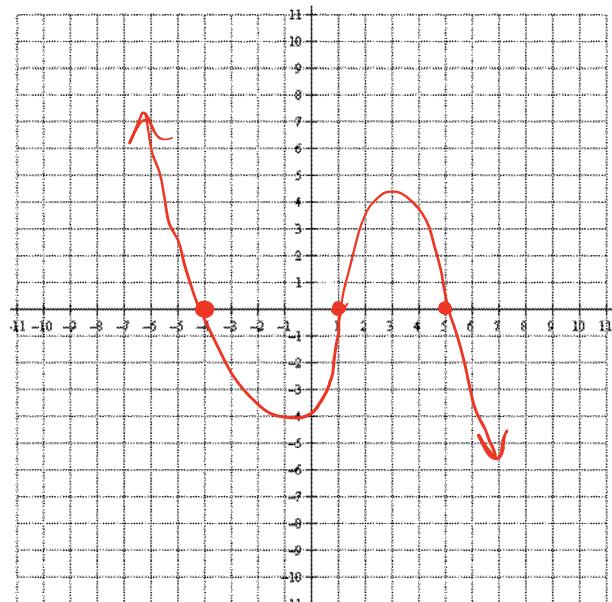


Notes 2.4 – Connections between $F(x)$ and $F'(x)$ for Polynomial and Trigonometric Functions

If $F'(x) \dots$	then $F(x) \dots$
...is = 0,	has a horizontal tangent line ∴ Likely has a relative max or min
...is > 0,	IS increasing
...is < 0,	IS decreasing
...changes from positive to negative,	Relative max
...changes from negative to positive,	relative Min

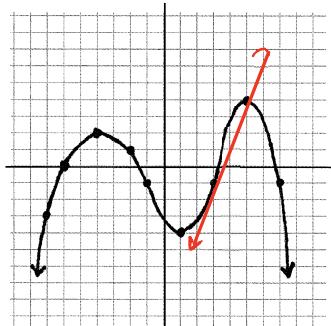


Graph of $f(x)$

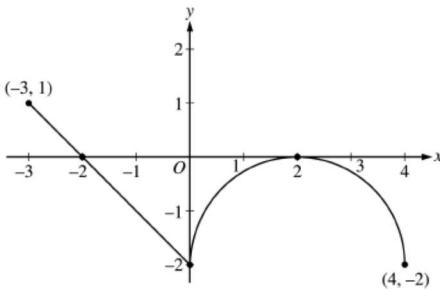


Possible Graph
of $f'(x)$

Pictured below is the graph of a function, $f(x)$. Answer the questions that follow about $f'(x)$.



Pictured below is the graph of $f'(x)$ on the interval $[-3, 4]$. Answer the following questions about $f(x)$.



Approximate the value of $f'(4)$.

Slope of Secant

$$f'(4) \approx \frac{f(5) - f(3)}{5 - 3} \approx \frac{4 - (-1)}{2} \approx \frac{5}{2}$$

On what open interval(s) is the graph of $f(x)$ increasing? Justify your reasoning.

$f(x)$ is increasing on $(-3, -2)$ because f' is above the x-axis

At what value(s) of x is $f'(x) = 0$. Justify your answer.

$f'(x) = 0$ at $x = -4, 1$, and 5 because $f(x)$ has relative maxes and mins at those locations

On what open interval(s) is the graph of $f(x)$ decreasing? Justify your answer.

$f(x)$ is decreasing on $(-2, 2) \cup (2, 4)$ because f' is below the x-axis

On what open interval(s) is $f'(x) < 0$? Justify your answer.

$f' < 0$ on $(-4, 1) \cup (5, \infty)$ because $f' < 0$ when $f(x)$ is decreasing.

At what value(s) of x does the graph of $f(x)$ have a horizontal tangent? Justify your answer.

$f(x)$ has a horizontal tangent at $x = -2, 2$ because $f'(x)$ is on the x-axis.

On what open interval(s) is $f'(x) > 0$? Justify your answer.

$f' > 0$ on $(-\infty, -4) \cup (1, 5)$ because $f' > 0$ when $f(x)$ is increasing

At what value(s) of x does the graph of $f(x)$ have a relative maximum? Justify your answer.

$f(x)$ has a relative maximum at $x = -2$ because f' goes from above to below the x-axis

At what value(s) of x does the graph of $f'(x)$ go from being below the x -axis to above the x -axis? Justify your answer.

f' goes from below to above the x-axis at $x = 1$ because $f(x)$ has a relative minimum at $x = 1$

At what value(s) of x does the graph of $f(x)$ have a relative minimum? Justify your answer.

$f(x)$ has no relative minimums because f' never goes from below to above the x-axis.

At what value(s) of x does the graph of $f'(x)$ go from being above the x -axis to below the x -axis? Justify your answer.

f' goes from above to below the x-axis at $x = -4, 5$ because $f(x)$ has a relative maximum at $x = -4, 5$.

What is the slope of the normal line to the graph of $f(x)$ at $x = 4$? Justify your reasoning.

The normal line's slope is $\frac{1}{2}$ at $x = 4$. The graph shows $f'(4) = -2$, which is the slope of the tangent line. The normal line is \perp to the tangent line.

Critical value = an x -value that changes the nature of a graph

Critical point = an ordered pair that changes the nature of a graph

- MIN
- MAX
- IP

For each of the given functions, determine the interval(s) on which $f(x)$ is increasing and/or decreasing. Find all coordinates of the relative extrema. Unless otherwise noted, perform the analysis on all values on $(-\infty, \infty)$. Provide justification for your answers.

1. $f(x) = 3x^5 - 5x^3$ ZON
 I) FIND CV by $f' = 0$ and $f' = \text{DNE}$
 II) FIND CP by $f''(c)$
 III) Draw f' sign diagram

I. CV: $x = -1, 0, 1$

$f'(x) = 15x^4 - 15x^2$	
$0 = 15x^2(x^2 - 1)$	
$0 = 15x^2(x-1)(x+1)$	
ZON	
$0 = 15x^2$	$x^2 - 1 = 0$
$0 = x^2$	$x^2 = 1$
$0 = x$	$x = \pm 1$

FACTORED

II. CP = $f''(c) : (-1, 2), (0, 0), (1, 2)$

$f(-1) = 3(-1)^5 - 5(-1)^3$
$= 3(-1) - 5(-1)$
$= -3 + 5$
$f(-1) = 2$
$f(0) = 3(0)^5 - 5(0)^3$
$f(0) = 0$
$f(1) = 3(1)^5 - 5(1)^3$
$= 3(1) - 5(1)$
$= 3 - 5$
$f(1) = -2$

$$f'(x) = 15x^2(x-1)(x+1)$$

III

$(-) (-) = +$	$(-) (+) = -$	$(-) (+) = -$	$(+) (+) = +$
$f' > 0$	$f' < 0$	$f' < 0$	$f' > 0$
$x = -2$	$x = -\frac{1}{2}$	$x = \frac{1}{2}$	$x = 2$
$f'(x) <$	$f'(x) = 0$	$f'(x) = 0$	$f'(x) > 0$

$f'(x)$

pos. MAX neg. NOT extreme neg. MIN pos.
 $(-1, 2)$ $(0, 1)$ $(1, 2)$

Answer: * $f(x)$ is increasing on $(-\infty, -1) \cup (1, \infty)$ because $f' > 0$ on that interval.

* $f(x)$ is decreasing on $(-1, 0) \cup (0, 1)$ because $f' < 0$ on that interval.

* $f(x)$ has a relative max at $(-1, 2)$ because $f'(x) > 0$ on the left and $f'(x) < 0$ on the right

* $f(x)$ has a relative min at $(1, -2)$ because $f' < 0$ on the left and $f' > 0$ on the right.

2. $f(\theta) = \theta + 2\sin\theta$ on $(0, 2\pi)$

CV: $\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

$f'(\theta) = 1 + 2\cos\theta$

$0 = 1 + 2\cos\theta$ ← Doesn't factor

$-1 = 2\cos\theta$

$-\frac{1}{2} = \cos\theta$ QUAD II, III $\frac{\pi}{3}$ reference

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$



CP: $(\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3}), (\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3})$

$$\begin{aligned}f\left(\frac{2\pi}{3}\right) &= \frac{2\pi}{3} + 2\sin\left(\frac{2\pi}{3}\right) \\&= \frac{2\pi}{3} + 2\left(\frac{\sqrt{3}}{2}\right)\end{aligned}$$

$$f\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} + \sqrt{3}$$

$$\begin{aligned}f\left(\frac{4\pi}{3}\right) &= \frac{4\pi}{3} + 2\sin\left(\frac{4\pi}{3}\right) \\&= \frac{4\pi}{3} + 2\left(-\frac{\sqrt{3}}{2}\right)\end{aligned}$$

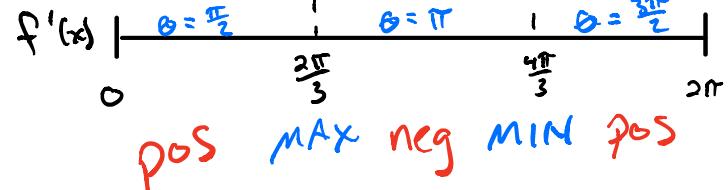
$$f\left(\frac{4\pi}{3}\right) = \frac{4\pi}{3} - \sqrt{3}$$

$f'(\theta) = 1 + 2\cos\theta$

$1 + 2\cos\frac{\pi}{2} =$

$1 + 2 \cdot 0 = +$

$f' > 0$



Answer: • $f(\theta)$ is increasing on $(0, \frac{2\pi}{3}) \cup (\frac{4\pi}{3}, 2\pi)$ because $f'(\theta) > 0$

• $f(\theta)$ is decreasing on $(\frac{2\pi}{3}, \frac{4\pi}{3})$ because $f'(\theta) < 0$

• $f(\theta)$ has a relative max at $(\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3})$ because $f'(\theta)$ changes from + to -.

• $f(\theta)$ has a relative min at $(\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3})$ because $f'(\theta)$ changes from - to +.