 Name______________________________

2017

AP® CollegeBoard

AP Calculus AB

Free-Response Questions

Name______________________________

+1 for proper units for (a), (c), (d)

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

2 3 5

horizontal cross section

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

(a) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure.

$V \approx 2(50.3) + 3(14.4) + 5(6.5)$ +1

$V \approx 176.3$ ft^3

+1

(b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

+1 A is a decreasing function with left Riemann Sum

$\therefore V \approx 173.3$ is an overestimate

(c) The area, in square feet, of the horizontal cross section at height h feet is modeled by the function f given by $f(h) = \dfrac{50.3}{e^{0.2h} + h}$. Based on this model, find the volume of the tank. Indicate units of measure.

$$V = \int_0^{10} f(h)\,dh \approx 101.325 \text{ ft}^3$$

+1 +1

$$Y_1 = \frac{50.3}{e^{.2X} + X}$$

$$\int_0^{10} (Y_1)\,dX$$

101.325338186

(d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

$$\left.\frac{dh}{dt}\right|_{h=5} = 0.26 \text{ ft/min}$$

$$V = \int_0^h f(x)\,dx$$

$$\frac{dV}{dt} = f(h) \cdot \frac{dh}{dt}$$

+1 +1

$$\left.\frac{dV}{dt}\right|_{h=5} = f(5) \cdot (0.26)$$

$Y_1(5)(.26)$

1.6944185624

$$\left.\frac{dV}{dt}\right|_{h=5} = 1.694 \text{ ft}^3/\text{min}$$

+1

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

Rate out

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \le 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

Rate IN

$$g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \le 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

(a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

$$\int_0^2 f(t)\,dt = 20.051$$

+1 +1

$\backslash Y_1 = 10 + .8X\sin(\frac{X^3}{100})$

$\backslash Y_2 = 3 + 2.4\ln(X^2 + 2X)$

$\int_0^2 (Y_1)dX$

20.0511751809

(b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

$f'(7) = -8.120 \frac{\text{pounds}}{\text{hr}^2}$ +1

$\frac{d}{dX}(Y_1)_{X=7}$

-8.119539823

At 7 hours, the rate at which bananas are removed from the display is decreasing by 8.120 pounds per hour each hour. +1

(c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

IN − OUT

$g(5) - f(5) = -2.263$

+1

$Y_2(5) - Y_1(5)$

-2.2631031298

$g(5) - f(5) < 0$ ∴ the number of pounds of bananas is decreasing.

+1

(d) How many pounds of bananas are on the display table at time $t = 8$?

pounds = Start + IN − OUT

+1 +1

$= 50 + \int_3^8 g(t)\,dt - \int_0^8 f(t)\,dt$

$= 50 + 58.964 - 85.617$

$\int_3^8 (Y_2)dX$

58.9640564045

$\int_0^8 (Y_1)dX$

85.6166606644

$50 + 58.964 - 85.617$

23.347

pounds on display = 23.347

+1

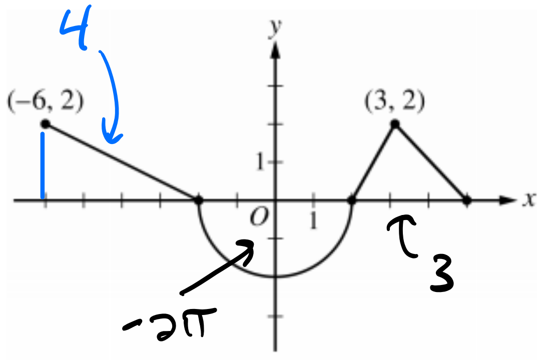
SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

$A = \pi r^2$
$= \pi 2^2$
$= 4\pi$



Graph of f'

f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

+1 FTC

$\int_{-6}^{-2} f'(x)dx = f(-2) - f(-6)$

$4 = 7 - f(-6)$

$-3 = -f(-6)$

$\boxed{3 = f(-6)}$ +1

$\int_{-2}^{5} f'(x)dx = f(5) - f(-2)$

$3 - 2\pi = f(5) - 7$

$\boxed{10 - 2\pi = f(5)}$ +1

(b) On what intervals is f increasing? Justify your answer.

$f' > 0$ on $[-6, -2) \cup (2, 5)$ +1

$\therefore f$ is increasing there. +1

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

$f' = 0$ at $x = -2$, $x = 2$, $x = 5$ +1

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

Part a → ($x = -6$)
Given → ($x = -2$)
Part a → ($x = 2$)

$$\int_{-2}^{2} f'(x)\,dx = f(2) - f(-2)$$

$$-2\pi = f(2) - 7$$

$$7 - 2\pi = f(2)$$

+1

The absolute minimum value on $[-6, 5]$ is $(7 - 2\pi)$

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

- $f''(-5) = \frac{2}{-4} = -\frac{1}{2}$ +1

- $f''(3)$ does not exist.
  b/c f' has a cusp at $x = 3$ +1

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

$H(0) = 91$

(a) $\boxed{\text{Write}}$ an equation for the line tangent to the graph of H at $t = 0$. $\boxed{\text{Use this equation}}$ to approximate the internal temperature of the potato at time $t = 3$.

Pot	SoT	Tangent
$(0,91)$	$\frac{dH}{dt} = -\frac{1}{4}(91-27)$ $= -16$ +1	$y - 91 = -16(x-0)$ +1

$$\begin{array}{r} 91 \\ -27 \\ \hline 64 \end{array}$$

$H(0) \approx -16(3) + 91$ +1

$H(0) \approx 43$

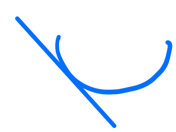
(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$. $\rightarrow$ $H = -16(3) + 91$

$= -48 + 91$

$= 43$

$$\begin{array}{r} 16 \\ 3 \\ \hline 48 \end{array} \qquad \begin{array}{r} 91 \\ -48 \\ \hline 43 \end{array}$$

$\frac{dH}{dt} = -\frac{1}{4}H + \frac{27}{4}$



$\frac{d^2H}{dt^2} = -\frac{1}{4}\frac{dH}{dt}$

$\frac{d^2H}{dt^2} = -\frac{1}{4}\left(-\frac{1}{4}H + \frac{27}{4}\right)$

$\left.\frac{d^2H}{dt^2}\right|_{\substack{t=3 \\ H=43}} = \underbrace{-\frac{1}{4}}_{NEG} \cdot \underbrace{\left(-\frac{1}{4}(43) + \frac{27}{4}\right)}_{NEG} > 0$ $\therefore$ H is concave up

$\therefore$ $H(0) \approx 43$ is an underestimate +1

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

Seperation of variables. . .

$$\frac{1}{(G-27)^{2/3}}\,dG = -dt \quad +1$$

$$\int (G-27)^{-2/3}\,dG = \int -\,dt$$

$u = G-27$
$du = dG$

$$\int u^{-2/3}\,du = -t + C$$

$$3u^{1/3} = -t + C$$

$$3(G-27)^{1/3} = -t + C \quad +1$$

@ $(0,91)$

$$3(91-27)^{1/3} = -(0) + C$$

$$3(64)^{1/3} = C$$

$$3(4) = C$$

$$12 = C \quad +1$$

$$3(G-27)^{1/3} = -t + 12$$

$$(G-27)^{1/3} = -\tfrac{1}{3}t + 4$$

$$G - 27 = \left(-\tfrac{1}{3}t + 4\right)^3$$

$$G = \left(-\tfrac{1}{3}t + 4\right)^3 + 27 \quad +1$$

$$G(3) = \left(-\tfrac{1}{3}(3) + 4\right)^3 + 27$$

$$G(3) = 27 + 27$$

$$G(3) = 54 \quad +1$$

5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$. $x_Q(0) = 5$

(a) For $0 \le t \le 8$, when is particle P moving to the left?

$v_P(t) = x_P'(t) = \dfrac{2t-2}{t^2-2t+10}$ +1

$v_P(t) = 0$

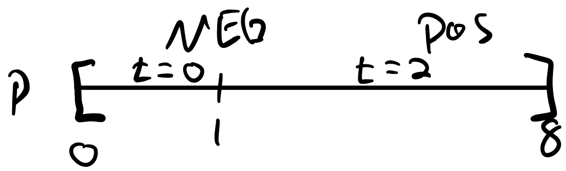
$2t - 2 = 0$

$t = 1$

$v_P(t)$ undefined

$t^2 - 2t + 10 = 0$

never

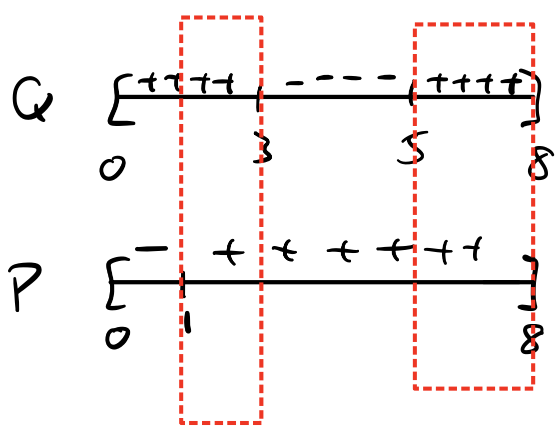


+1

P is moving left on $[0, 1)$

(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

$v_Q(t) = (t-3)(t-5)$



+1

Both Q and P move right during $(1, 3)$ and $(5, 8]$.

+1

(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

$a_Q(t) = v_Q'(t) = 2t - 8$

$a_Q(2) = 2(2) - 8$

$a_Q(2) = -4$ +1

$v_Q(2) = (2)^2 - 8(2) + 15$

$= 4 - 16 + 15$

$v_Q(2) = 3$

At $t = 2$, the acceleration and velocity are opposite signs.

$\therefore$ Q's speed is decreasing. +1

(d) Find the position of particle Q the first time it changes direction.

From (b), Q first changes direction at $t = 3$.

$x_Q(t) = \int (t^2 - 8t + 15)dt$

$x_Q(t) = \frac{1}{3}t^3 - 4t^2 + 15t + C$ +1

@(0,5) $5 = \frac{1}{3}(0)^3 - 4(0)^2 + 15(0) + C$

$5 = C$

+1

$x_Q(t) = \frac{1}{3}t^3 - 4t^2 + 15t + 5$

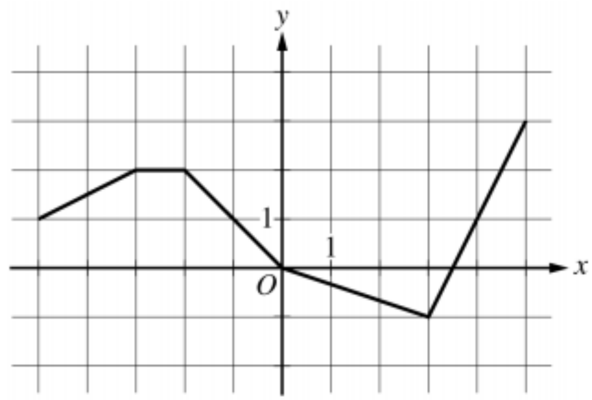
$x_Q(3) = \frac{1}{3}(3)^3 - 4(3)^2 + 15(3) + 5$

$= 9 - 36 + 45 + 5$

$= -27 + 50$

$x_Q(3) = 23$ +1

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

$f'(x) = -2\sin(2x) + \cos x \cdot e^{\sin x}$

$f'(\pi) = -2\sin(2\pi) + \cos(\pi)\, e^{\sin(\pi)}$

$= -2 \cdot 0 + (-1) \cdot e^{0}$

$f'(\pi) = -1$ +1 +1

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

$k'(x) = h'(f(x)) \cdot f'(x)$ +1

$k'(\pi) = h'(f(\pi)) \cdot f'(\pi)$

$= h'(2) \cdot (-1)$

$= -\frac{1}{3}(-1)$

$k'(\pi) = \frac{1}{3}$ +1

$f(\pi) = \cos(2\pi) + e^{\sin(\pi)}$

$= 1 + e^{0}$

$f(\pi) = 2$

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

$$m'(x) = g'(-2x)(-2) \cdot h(x) + g(-2x) \cdot h'(x)$$ +1 +1

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$$m'(2) = g'(-2 \cdot 2)(-2) \cdot h(2) + g(-2 \cdot 2) \cdot h'(2)$$

$$= (-1)(-2)\left(-\tfrac{2}{3}\right) + 5 \cdot \left(-\tfrac{1}{3}\right)$$ +1

$$= -\frac{4}{3} - \frac{5}{3}$$

$$= -\frac{9}{3}$$

$$m'(2) = -3$$

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

g is differentiable $\therefore$ g is continuous.

$$\text{ARC} = \frac{g(-5) - g(-3)}{-5-(-3)} = \frac{10-2}{-2} = \frac{8}{-2} = -4$$ +1

By MVT there exists a "c" on $[-5,-3]$ such that $g'(c) = -4$. +1