2.1 Average Rate of Change

Test Prep

1. The cost of producing x units of a certain item is $c(x) = 2{,}000 + 8.6x + 0.5x^2$. What is the average rate of change of c with respect to x when the level of production increases from $x = 300$ to $x = 310$ units?

(A) 313.6 (B) 310 (C) 214.2 (D) 200 (E) 10

$$ARC = \frac{\Delta y}{\Delta x} = \frac{C(310) - C(300)}{310 - 300} = \frac{(52{,}716) - (49580)}{10} = \frac{3136}{10}$$

$$ARC = 313.6$$

Calculator

$c(310) = 52{,}716$

$c(300) = 49{,}580$

2. Which of the following is true of the function $f(x) = \sqrt{x^2+1}$?

(A) $\lim\limits_{x\to\infty}(f(x) - x) = 0$ and $\lim\limits_{x\to-\infty}(f(x) - x) = 0$ (B) $\lim\limits_{x\to\infty}(f(x) + x) = 0$ and $\lim\limits_{x\to-\infty}(f(x) - x) = 0$

(C) $\lim\limits_{x\to\infty}(f(x) - x) = 0$ and $\lim\limits_{x\to-\infty}(f(x) + x) = 0$ (D) $\lim\limits_{x\to\infty}(f(x) + x) = 0$ and $\lim\limits_{x\to-\infty}(f(x) + x) = 0$

(E) None of the above

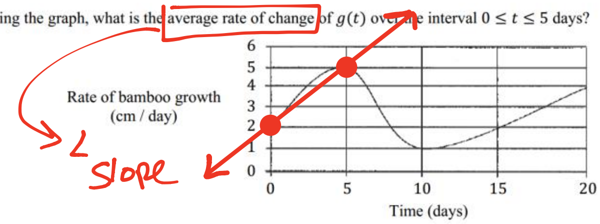
$$\lim_{x\to\infty}\left(\sqrt{x^2+1} - x\right)\frac{\left(\sqrt{x^2+1} + x\right)}{\left(\sqrt{x^2+1} + x\right)} = \lim_{x\to\infty}\frac{x^2+1-x^2}{\sqrt{x^2+1}+x} = \lim_{x\to\infty}\frac{1}{\sqrt{x^2+1}+x} = 0$$

$\boxed{\text{recall } \frac{1}{\infty} = 0}$

$$\lim_{x\to-\infty}\left(\sqrt{x^2+1} + x\right)\frac{\left(\sqrt{x^2+1} - x\right)}{\left(\sqrt{x^2+1} - x\right)} = \lim_{x\to-\infty}\frac{x^2+1-x^2}{\sqrt{x^2+1}-x} = \lim_{x\to-\infty}\frac{1}{\sqrt{x^2+1}-x} = 0$$

$\boxed{\text{recall } \frac{1}{-\infty} = 0}$

3. Using the graph, what is the $\boxed{\text{average rate of change}}$ of $g(t)$ over the interval $0 \le t \le 5$ days?



(A) $\frac{3}{5}$ cm per day per day (B) 1 cm per day per day (C) $\frac{7}{5}$ cm per day per day

(D) 3 cm per day per day (E) $\frac{7}{2}$ cm per day per day

$$ARC = \frac{\Delta y}{\Delta x}\frac{\text{cm}}{\text{day}} = \frac{(5)-(2)}{(5)-(0)}$$

$$ARC = \frac{3 \text{ cm}}{5 \text{ day}}$$

4. Which of the following functions has a $\boxed{\text{vertical asymptote at } x = 4}$?

(A) $\frac{x+5}{x^2-4}$ (B) $\frac{x^2-16}{x-4}$ (C) $\frac{4x}{x+1}$ (D) $\frac{x+6}{x^2-7x+12}$ (E) None of the above

(A) $\frac{x+5}{(x-2)(x+2)}$ VA @ $x = \pm 2$

(B) $\frac{(x+4)\cancel{(x-4)}}{\cancel{(x-4)}}$ NO VA

(C) VA @ $x = -1$

(D) $\frac{x+6}{(x-4)(x-3)}$ VA @ $x = 3, 4$

5. A tank holds 10,000 liters of gasoline. At the bottom of the tank, a lever can be turned to allow the gasoline to be dispensed. The tank can be emptied in exactly 40 minutes. Below is a table which gives the volume v of gasoline (in liters) which remain in the tank after t minutes of draining have taken place.

t (minutes)	0	5	10	15	20	25	30	35	40
v (liters)	4700	4100	3200	2400	2000	1400	800	500	0

During which of the following 10-minute intervals is the average rate of gasoline draining from the tank the $\boxed{\text{least}}$?

(A) $t = 0$ to $t = 10$ minutes (B) $t = 10$ to $t = 20$ minutes (C) $t = 15$ to $t = 25$ minutes

(D) $t = 25$ to $t = 35$ minutes (E) $t = 30$ to $t = 40$ minutes

$0 \text{ to } 10 = \frac{4700-3200}{10} = \frac{1500}{10}$

$10 \text{ to } 20 = \frac{3200-2000}{10} = \frac{1200}{10}$

$15 \text{ to } 25 = \frac{2400-1400}{10} = \frac{1000}{10}$

$25 \text{ to } 35 = \frac{1400-500}{10} = \frac{900}{10}$

$30 \text{ to } 40 = \frac{800-0}{10} = \frac{800}{10}$

2.2 Definition of the Derivative

Test Prep

1. Let $f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$. For what value of x does $f(x) = 4$?

(A) -4 (B) -1 (C) 1 (D) 2 (E) 4

$f(x) = x^2$

$4 = x^2$

$\pm 2 = x$

2. If $f(x+y) = f(x)\cdot f(y)$ and if $\lim_{h\to 0} \frac{f(h)-1}{h} = 6$, then $f'(x) =$

(A) 6 (B) $6 + f(x)$ (C) $6 \cdot f(x)$

(D) $6 + f(h)$ (E) $6 \cdot f(h)$

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3. Which of the following gives the derivative of the function $f(x) = x^2$ at the point $(2, 4)$?

(A) $\lim_{h\to 0} \frac{(x+2)^2 - x^2}{4}$ (B) $\lim_{h\to \infty} \frac{(2+h)^2 - 2^2}{h}$ (C) $\frac{(2+h)^2 - 2^2}{h}$

(D) $\lim_{h\to 0} \frac{(2+h)^2 - 2^2}{h}$ (E) $\lim_{h\to 0} \frac{(4+h)^2 - 4^2}{h}$

$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

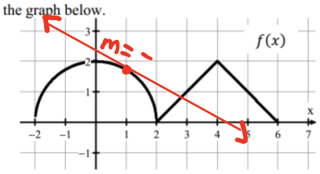
$@x=2: \lim_{h\to 0} \frac{(2+h)^2 - (2)^2}{h}$

2.3 Differentiability **Test Prep**

1. f is continuous for $a \le x \le b$ but not differential for some c such that $a < c < b$. Which of the following could be true?

(A) $x = c$ is a vertical asymptote of the graph of f.
(B) $\lim_{x \to c} f(x) \ne f(c)$
(C) The graph of f has a cusp at $x = c$.
(D) $f(c)$ is undefined.
(E) None of the above

Questions 2 and 3 refer to the graph below.

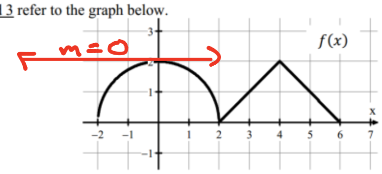


2. The graph of $f(x)$, shown above, consists of a semicircle and two line segments. $f'(1) =$

(A) -1 (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) 1 (E) $\sqrt{3}$

$-\frac{1}{1}$ $\frac{-1}{1.732}$

Questions 2 and 3 refer to the graph below.



3. For which values of x does $f'(x) = 0$? Tangent line has Slope of zero.

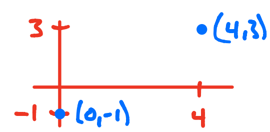
(A) 0 only (B) 2 only (C) 0 and 4 only
(D) $-2, 2,$ and 6 only (E) $-2, 0, 2, 4,$ and 6

4. If f is a differentiable function and $f(0) = -1$ and $f(4) = 3$, then which of the following must be true?

I. There exists a c in $[0,4]$ where $f(c) = 0$. IVT
II. There exists a c in $[0,4]$ where $\boxed{f'(c) = 0.}$ means a max or min.
III. There exists a c in $[0,4]$ where $f'(c) = 1$. MVT

(A) I only (B) II only (C) I and II only
(D) I and III only (E) I, II, and III

INTEMEDIATE VALUE Theorem



Mean Value Theorem: ARC = $\frac{(3)-(-1)}{(4)-(0)}$

ARC = $\frac{4}{4}$

ARC = 1

∴ $f'(c) = 1$ must exist

5. If $f'(x) = \tan^{-1}(x^3 - x)$, at how many points is the $\boxed{\text{tangent line to the graph of } y = f(x)}$ parallel to the line $y = 2x$?

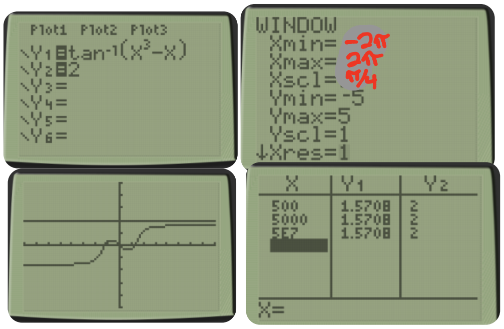
$f'(x)$

$m=2$

$1/m=2$

(A) None (B) One (C) Two (D) Three (E) Infinitely many

$\tan^{-1}(x^3 - x) = 2$



6. $\lim_{x \to 0} \frac{\sin^3(3x)}{x^3} = \lim_{x\to 0}\left[\frac{3\cdot\sin(3x)}{3\cdot x}\right]^3 = 3^3 \lim_{x\to 0}\left[\frac{\sin(3x)}{3x}\right]^3 = 27 \cdot 1$

$\boxed{\text{Think: } \lim_{x\to 0} \frac{\sin x}{x} = 1}$

(A) 0 (B) 1 (C) 3 (D) 9 (E) 27

or

$\lim_{x\to 0}\left[\frac{3\cdot\sin(3x)}{3\cdot x}\right]^3 = \lim_{x\to 0}\left[3\,\frac{\sin(3x)}{3x}\right]^3 = [3\cdot 1]^3 = [3]^3 = 27$