

Part III: Integrate the trig functions by substitution when needed

$$\#1) \quad \int 3 \cos(4x) dx$$

$$= \int 3 \cos(u) \left(\frac{du}{4}\right)$$

$$= \frac{3}{4} \int \cos(u) du$$

$$= \frac{3}{4} \sin(u) + C$$

$$= \frac{3}{4} \sin(4x) + C$$

$$u = 4x$$

$$\frac{du}{dx} = 4$$

$$du = 4dx$$

$$\frac{du}{4} = dx$$

$$u = 4x$$

$$\frac{du}{dx} = 4$$

$$du = 4dx$$

$$\frac{du}{4} = dx$$

$$\#3) \quad \int 4 \sec(4x) \tan(4x) dx$$

$$= \int 4 \sec(u) \tan(u) \left(\frac{du}{4}\right)$$

$$= \int \sec(u) \tan(u) du$$

$$= \sec(u) + C$$

$$= \sec(4x) + C$$

$$\#2) \quad \int \sin(4x) dx$$

$$= \int \sin(u) \frac{du}{4}$$

$$= \frac{1}{4} \int \sin(u) du$$

$$= -\frac{1}{4} \cos(u) + C$$

$$= -\frac{1}{4} \cos(4x) + C$$

$$u = 4x$$

$$\frac{du}{dx} = 4$$

$$du = 4dx$$

$$\frac{du}{4} = dx$$

$$\#4) \quad \int \cos(8x) dx$$

$$= \int \cos(u) \left(\frac{du}{8}\right)$$

$$= \frac{1}{8} \int \cos(u) du$$

$$= \frac{1}{8} \sin(u) + C$$

$$= \frac{1}{8} \sin(8x) + C$$

$$u = 8x$$

$$\frac{du}{dx} = 8$$

$$du = 8dx$$

$$\frac{du}{8} = dx$$

$$\#5) \quad \int \cos(11x) \sqrt{\sin(11x)} dx$$

$$\begin{aligned}
 &= \int \cos(11x) u^{\frac{1}{2}} \frac{du}{11\cos(11x)} \quad \left| \begin{array}{l} u = \sin(11x) \\ du = 11 \cdot \cos(11x) dx \end{array} \right. \\
 &= \frac{1}{11} \int u^{\frac{1}{2}} du \quad \left| \begin{array}{l} \frac{du}{11\cos(11x)} = dx \\ \end{array} \right. \\
 &= \frac{1}{11} \cdot \frac{2}{3} u^{\frac{3}{2}} + C \\
 &= \frac{2}{33} \sqrt{\sin^3(11x)} + C
 \end{aligned}$$

$$\#6) \quad \int \sin\left(\frac{\pi}{3}x\right) dx$$

$$\begin{aligned}
 &= \int \sin(u) \frac{3}{\pi} du \quad \left| \begin{array}{l} u = \frac{\pi}{3}x \\ du = \frac{\pi}{3} dx \\ \frac{3}{\pi} du = dx \end{array} \right. \\
 &= \frac{3}{\pi} \int \sin(u) du \\
 &= \frac{3}{\pi} (-\cos(u)) + C \\
 &= -\frac{3}{\pi} \cos\left(\frac{\pi}{3}x\right) + C
 \end{aligned}$$

$$\#7) \quad \int \csc(x) \cot(x) \csc^2(x) dx$$

$$\begin{aligned}
 &= \int \csc(x) \cancel{\cot(x)} u \cdot \frac{du}{-\cot(x)} \quad \left| \begin{array}{l} u = \csc^2(x) \\ du = -\cot(x) dx \\ \frac{du}{-\cot(x)} = dx \end{array} \right. \\
 &= - \int \csc(x) \cdot u du \\
 &= - \int \sqrt{u} u du \\
 &= - \int u^{\frac{3}{2}} du \\
 &= -\frac{2}{5} u^{\frac{5}{2}} + C \\
 &= -\frac{2}{5} (\sqrt{\csc^2(x)})^5 + C \\
 &= -\frac{2}{5} \csc^5(x) + C
 \end{aligned}$$

$$\#8) \quad \int \frac{\sec(x)}{\cos(x)} dx$$

$$\begin{aligned}
 &= \int \sec^2(x) dx \\
 &= \tan(x) + C
 \end{aligned}$$

#9) $\int 2x \cos(x^2) dx$

$$\begin{aligned} &= \int 2x \cos(u) \left(\frac{du}{2x} \right) \\ &= \int \cos(u) du \\ &= \sin(u) + C \\ &= \sin(x^2) + C \end{aligned}$$

$$\begin{aligned} u &= x^2 \\ \frac{du}{dx} &= 2x \\ du &= 2x dx \\ \frac{du}{2x} &= dx \end{aligned}$$

#10) $\int \sin(4x) dx$

$$\begin{aligned} &= \int \sin(u) \left(\frac{du}{4} \right) \\ &= \frac{1}{4} \int \sin(u) du \\ &= -\frac{1}{4} \cos(u) + C \\ &= -\frac{1}{4} \cos(4x) + C \end{aligned}$$

$$\begin{aligned} u &= 4x \\ \frac{du}{dx} &= 4 \\ du &= 4 dx \\ \frac{du}{4} &= dx \end{aligned}$$

#11) $\int 3x^2 \sec(x^3) \tan(x^3) dx$

$$\begin{aligned} &= \int 3x^2 \sec(u) \tan(u) \left(\frac{du}{3x^2} \right) \\ &= \int \sec(u) \tan(u) du \\ &= \sec(u) + C \\ &= \sec(x^3) + C \end{aligned}$$

$$\begin{aligned} u &= x^3 \\ \frac{du}{dx} &= 3x^2 \\ du &= 3x^2 dx \\ \frac{du}{3x^2} &= dx \end{aligned}$$

#12) $\int \frac{1}{27} \sin(27x) dx$

$$\begin{aligned} &= \int \frac{1}{27} \sin(u) \left(\frac{du}{27} \right) \\ &= \frac{1}{27} \int \sin(u) du \\ &= -\frac{1}{27} \cos(u) + C \\ &= -\frac{1}{27} \cos(27x) + C \end{aligned}$$

$$\begin{aligned} u &= 27x \\ \frac{du}{dx} &= 27 \\ du &= 27 dx \\ \frac{du}{27} &= dx \end{aligned}$$

#13) $\int \sec^2(x) \sqrt{\tan(x)} dx$

$$\begin{aligned} &= \int \sec^2(u) \sqrt{u} \left(\frac{du}{\sec^2(u)} \right) \\ &= \int u^{1/2} du \\ &= \frac{2}{3} u^{3/2} + C \\ &= \frac{2}{3} \sqrt{\tan^3(x)} + C \end{aligned}$$

$$\begin{aligned} u &= \tan(x) \\ \frac{du}{dx} &= \sec^2(x) \\ du &= \sec^2(x) dx \\ \frac{du}{\sec^2(u)} &= dx \end{aligned}$$

#14) $\int \frac{1}{\pi} \csc^2(\pi x) dx$

$$\begin{aligned} &= \int \frac{1}{\pi} \csc^2(u) \left(\frac{du}{\pi} \right) \\ &= \frac{1}{\pi^2} \int \csc^2(u) du \\ &= \frac{1}{\pi^2} (-\cot(u)) + C \\ &= -\frac{1}{\pi^2} \cot(\pi x) + C \end{aligned}$$

$$\begin{aligned} u &= \pi x \\ \frac{du}{dx} &= \pi \\ du &= \pi dx \\ \frac{du}{\pi} &= dx \end{aligned}$$

$$\#15) \int \tan(x) \ln |\sec(x)| dx$$

$$= \int \tan(x) \cdot u \left(\frac{du}{\tan(x)} \right)$$

$$= \int u du$$

$$= \frac{1}{2} u^2 + C$$

$$= \frac{1}{2} \left[\ln |\sec(x)| \right]^2 + C$$

$$u = \ln |\sec(x)|$$

$$\frac{du}{dx} = \frac{\sec'(x)}{\sec(x)}$$

$$\frac{du}{dx} = \frac{\sec(x) \cdot \tan(x)}{\sec(x)}$$

$$\frac{du}{dx} = \tan(x)$$

$$du = \tan(x) dx$$

$$\frac{du}{\tan(x)} = dx$$

$$\#16) \int \sec(x) \ln |\sec(x) + \tan(x)| dx$$

$$= \int \sec(x) \cdot u \left(\frac{du}{\sec(x)} \right)$$

$$= \int u du$$

$$= \frac{1}{2} u^2 + C$$

$$= \frac{1}{2} \left[\ln |\sec(x) + \tan(x)| \right]^2 + C$$

$$u = \ln |\sec(x) + \tan(x)|$$

$$\frac{du}{dx} = \sec(x)$$

$$du = \sec(x) dx$$

$$\frac{du}{\sec(x)} = dx$$

17. $\int \frac{2x^2}{\sqrt{x^3-2}} dx =$

- $\int \frac{2x^2}{\sqrt{u}} \frac{du}{3x^2}$ (A) $\frac{4}{3}(x^3-2)^{1/2} + C$
 B. $\frac{1}{3}(x^3-2)^{1/2} + C$
 $\frac{2}{3} \int u^{-1/2} du$ C. $\frac{2}{3}(x^3-2)^{1/2} + C$
 $\frac{4}{3} u^{1/2} + C$ D. $2(x^3-2)^{1/2} + C$
 E. $3(x^3-2)^{1/2} + C$

$$u = x^3 - 2$$

$$\frac{du}{dx} = 3x^2$$

$$du = 3x^2 dx$$

$$\frac{du}{3x^2} = dx$$

18. If $u = 2x - 3$, then $\int x^3 \sqrt{2x-3} dx =$

- $u+3=2x \Rightarrow x = \frac{1}{2}(u+3)$ (A) $\frac{1}{4} \int u^{4/3} + 3u^{1/3} du$
 B. $\frac{1}{2} \int u^{2/3} + 3u^{4/3} du$
 C. $\frac{1}{2} \int \sqrt[3]{u} du$
 D. $\frac{1}{4} \int \sqrt[3]{u} du$
 E. $\frac{1}{2} \int (2u+3) \sqrt[3]{u} du$

$$\int \frac{1}{2}(u+3)^3 \sqrt{u} \frac{1}{2} du$$

$$u = 2x - 3$$

$$\frac{du}{dx} = 2$$

$$\frac{1}{2} du = dx$$

19. If $\frac{dy}{dx} = \frac{x^2}{y}$ and $f(0) = -4$, find the particular solution to the differential equation. (See next section)

A. $f(x) = \frac{1}{3}x^3 + 4$

B. $f(x) = -\sqrt{\frac{2}{3}x^3 + 16}$

C. $f(x) = \sqrt{\frac{2}{3}x^3 + 16}$

D. $f(x) = \frac{1}{3}x^3$

E. $f(x) = -\sqrt{\frac{2}{3}x^3 + 8}$

$$\int y dy = \int x^2 dx$$

$$\frac{1}{2}y^2 = \frac{1}{3}x^3 + C$$

at (0, -4)
 $\frac{1}{2}(-4)^2 = \frac{1}{3}(0)^3 + C$

$$\frac{1}{2}(16) = C$$

$$8 = C$$

$$\frac{1}{2}y^2 = \frac{1}{3}x^3 + 8$$

$$y^2 = \frac{2}{3}x^3 + 16$$

$$y = \pm \sqrt{\frac{2}{3}x^3 + 16}$$

which equation contains (0, -4)?

20. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ is equal to which of the following?

A. $2 \int_1^{16} e^u du$

B. $2 \int_1^4 e^u du$

C. $2 \int_1^2 e^u du$

D. $\frac{1}{2} \int_1^2 e^u du$

E. $\int_1^4 e^u du$

$$\int_1^4 \frac{e^u}{\sqrt{x}} 2\sqrt{x} du$$

$$u = x^{1/2}$$

$$du = \frac{1}{2}x^{-1/2} dx$$

$$2\sqrt{x} du = dx$$

21. $\int_0^1 e^{-4x} dx =$

A. $-\frac{e^{-4}}{4}$

B. $-4e^{-4}$

C. $e^{-4} - 1$

D. $\frac{1}{4} - \frac{e^{-4}}{4}$

E. $4 - 4e^{-4}$

$$\int e^u (-\frac{1}{4} du)$$

$$-\frac{1}{4} \int e^u du$$

$$\frac{1}{4} \int e^u du$$

$$\frac{1}{4} - \frac{e^{-4}}{4}$$

$$u = -4x$$

$$du = -4 dx$$

$$-\frac{1}{4} du = dx$$

22. $\int \frac{x}{x^2-4} dx = \frac{1}{2} \int \frac{1}{u} du$

$u = x^2 - 4$

$du = 2x dx$

$\frac{1}{2} du = x dx$

A. $\frac{-1}{4(x^2-4)^2} + C$

B. $\frac{1}{2(x^2-4)} + C$

C. $\frac{1}{2} \ln|x^2-4| + C$

D. $2 \ln|x^2-4| + C$

E. $\frac{2}{x^2-4} + C$

$$= \frac{1}{2} \ln|u| + C$$

$$= \frac{1}{2} \ln|x^2-4| + C$$