4.1 The function $C(x)$ gives the cost of digging a hole x feet deep.

$\mathbf{C(20) = 140}$ means that a hole ___20 feet_____ deep costs ___\$140___ to dig.

$\mathbf{C'(20) = 5}$ means that when the hole is _____20 feet deep______, the cost of digging is _increasing____ at a rate of ___\$5 per foot_____________.

4.2 A particle moves back and forth on a horizontal track for $0 \le t < \frac{\pi}{2}$ minutes. The particle's position, in feet, is given by the function $s(t) = \frac{1}{2}\tan t$. Find the acceleration of the particle at time $t = \frac{\pi}{6}$ minutes and indicate units of measure.

$$v(t) = s'(t) = \tfrac{1}{2}\sec^2 t = \tfrac{1}{2}(\sec t)^2$$

$$a(t) = v'(t)' = s''(t) = 2 \cdot \frac{1}{2}(\sec t)\sec t \tan t$$

$$a\left(\frac{\pi}{6}\right) = \sec^2\frac{\pi}{6}\tan\frac{\pi}{6} = \left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{\sqrt{3}}{3}\right) = \frac{4}{3\sqrt{3}} = \frac{4\sqrt{3}}{9}\text{ feet per min per min (ft/min}^2\text{)}$$

4.3 If $P(t)$ models the size of a population at time $t > 0$, which of the following differential equations describes linear growth in the size of the population? Which describes exponential growth?

$\frac{dP}{dt} = 200$ Linear Growth	$\frac{dP}{dt} = 200t$	$\frac{dP}{dt} = 100t^2$	$\frac{dP}{dt} = 200P$ Exponential Growth	$\frac{dP}{dt} = 100P^2$

4.4 Determine $\frac{dz}{dt}$ if you know that $z = xy^2,\ z = 3,\ y = \frac{1}{2},\ \frac{dx}{dt} = -2,$ and $\frac{dy}{dt} = 5$.

$$\frac{dz}{dt} = y^2\frac{dx}{dt} + 2xy\frac{dy}{dt}$$

$$3 = x\left(\frac{1}{2}\right)^2 \qquad \left.\frac{dz}{dt}\right|_{\left(12,\frac{1}{2},3\right)} = \left(\frac{1}{2}\right)^2(-2) + 2(12)\left(\frac{1}{2}\right)(5)$$

$$3 = \frac{x}{4} \qquad = -\frac{1}{2} + 60$$

$$x = 12 \qquad = 59\frac{1}{2}$$

4.5 Free Response Question (FRQ) Practice in a bit!!

4.6 Given $g(x)$ is a differentiable function about which little else is known other that $g(-3) = 2$ and $g'(-3) = 7$. Use the tangent line of $g(x)$ at $x = -3$ to approximate $g(-2.9)$.

$$y = 2 + 7(x+3) \qquad g(-2.9) \approx 2 + 7(-2.9+3) = 2 + 0.7 = 2.7$$

4.7 $\lim_{x\to5}\frac{x^4-625}{x^2-25}$ $\quad \lim_{x\to5}(x^4 - 625) = 0 = \lim_{x\to5}(x^2 - 25)$ $\quad$ – OR – $\quad \lim_{x\to5}\frac{(x^2+25(x^2-25)}{x^2-25}$

$$\therefore \lim_{x\to5}\frac{x^4-625}{x^2-25} = \lim_{x\to5}\frac{4x^3}{2x} = \lim_{x\to5}(2x^2) = 2(5)^2 = 50 \qquad \lim_{x\to5}(x^2+25) = 50$$

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Question 4

(a) $V = \pi r^2 h = \pi(1)^2 h = \pi h$ $\left.\frac{dV}{dt}\right\|_{h=4} = \pi \left.\frac{dh}{dt}\right\|_{h=4} = \pi\left(-\frac{1}{10}\sqrt{4}\right) = -\frac{\pi}{5}$ cubic feet per second	$2 : \begin{cases} 1 : \frac{dV}{dt} = \pi \frac{dh}{dt} \\ 1 : \text{answer with units} \end{cases}$
(b) $\frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} = -\frac{1}{20\sqrt{h}} \cdot \left(-\frac{1}{10}\sqrt{h}\right) = \frac{1}{200}$ Because $\frac{d^2h}{dt^2} = \frac{1}{200} > 0$ for $h > 0$, the rate of change of the height is increasing when the height of the water is 3 feet.	$3 : \begin{cases} 1 : \frac{d}{dh}\left(-\frac{1}{10}\sqrt{h}\right) = -\frac{1}{20\sqrt{h}} \\ 1 : \frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} \\ 1 : \text{answer with explanation} \end{cases}$

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Question 4

(d) $G(x) = 50 \Rightarrow x = 1$ $\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1+x)100 - 100x \cdot 1}{(1+x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1+x)^2} \cdot \frac{dx}{dt}$ $\left.\frac{d}{dt}(G(x))\right\|_{x=1} = \frac{100}{(1+1)^2} \cdot 0.03 = \frac{3}{4}$ According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\frac{3}{4}$ meter per year.	$3 : \begin{cases} 2 : \frac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$ Note: max 1/3 [1-0] if no chain rule

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Question 4

(a) $H'(0) = -\frac{1}{4}(91 - 27) = -16$

$H(0) = 91$

An equation for the tangent line is $y = 91 - 16t$.

The internal temperature of the potato at time $t = 3$ minutes is approximately $91 - 16 \cdot 3 = 43$ degrees Celsius.

$3 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$

(b) $\frac{d^2H}{dt^2} = -\frac{1}{4}\frac{dH}{dt} = \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)(H - 27) = \frac{1}{16}(H - 27)$

$H > 27$ for $t > 0 \Rightarrow \frac{d^2H}{dt^2} = \frac{1}{16}(H - 27) > 0$ for $t > 0$

Therefore, the graph of H is concave up for $t > 0$. Thus, the answer in part (a) is an underestimate.

1 : underestimate with reason

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Question 5

(c) $\frac{dr}{dt} = \frac{1}{20}(2h)\frac{dh}{dt}$

$-\frac{1}{5} = \frac{3}{10}\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{1}{5} \cdot \frac{10}{3} = -\frac{2}{3}$ in/sec

$3 : \begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{dr}{dt} = \frac{1}{20}(2h)\frac{dh}{dt}$

$-\frac{1}{5} = \frac{3}{10}\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{1}{5} \cdot \frac{10}{3} = -\frac{2}{3}$ in/sec

$3 : \begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

Question 4

(d) Let x be train A's position, y train B's position, and z the distance between train A and train B. $z^2 = x^2 + y^2 \Rightarrow 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$ $x = 300,\ y = 400 \Rightarrow z = 500$ $v_B(2) = -20 + 120 + 25 = 125$ $500\frac{dz}{dt} = (300)(100) + (400)(125)$ $\frac{dz}{dt} = \frac{80000}{500} = 160$ meters per minute	3 : { 2 : implicit differentiation of distance relationship 1 : answer