Unit 1 - 4 – L'Hospital's Rule

When the ratio of two functions tends to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ in the limits, such forms are said to be indeterminant. There are other indeterminant forms such as $\infty - \infty$ which you were learn how to handle in subsequent math courses.

If functions f and g are differentiable on an open interval I, except for perhaps at $x = a$, and if $\lim\limits_{x\to a} f(x) = 0 = \lim\limits_{x\to a} g(x)$, then $\lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$. If necessary, rinse and repeat.

The same holds true for limits whose numerator and denominator both approach $\pm$infinity as x approaches a.

Evaluate the limits. Use L'Hospital's Rule only if it applies.

1. $\lim\limits_{x\to 0}\frac{\sin(2x)}{\sin(5x)}$	2. $\lim\limits_{x\to 0}\frac{\sqrt{x+1}-1-x/2}{5x^2}$
3. $\lim\limits_{x\to\infty}\frac{\ln x}{4\sqrt{x}}$	4. Note: $f(1) = 1,\ f'(1) = 2$ and $f''(1) = 3$ $\lim\limits_{x\to 1}\frac{10^x-3f'(x)}{f(x)-\arctan x}$

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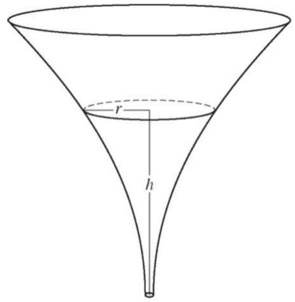

5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

   (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

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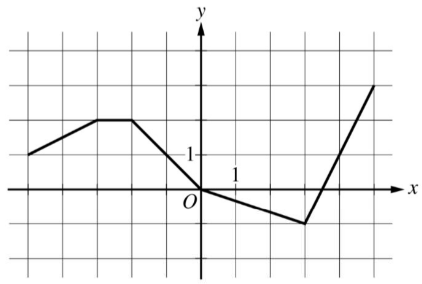
x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x. Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c)Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.