Section I: Multiple Choice

The multiple-choice section on each exam is designed for broad coverage of the course content. Multiple-choice questions are discrete, as opposed to appearing in question sets, and the questions do not appear in the order in which topics are addressed in the curriculum framework. There are 30 multiple-choice questions in Part A and 15 multiple-choice questions in Part B; students may use a graphing calculator only for Part B. Each part of the multiple-choice section is timed and students may not return to questions in Part A of the multiple-choice section once they have begun Part B.

Curriculum Framework Alignment and Rationales for Answers

Question 1

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.3B(b) Evaluate definite integrals.	**3.3B2:** If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 5:** Building notational fluency

(A)	This option is incorrect. Lack of parentheses is a common mistake, resulting in sign errors, when the antiderivative consists of more than one term: $\frac{x^3}{3} - x^2\Big\vert_{-1}^{3} = \frac{27}{3} - 9 - \left(-\frac{1}{3}\right) - 1 = 0 + \frac{1}{3} - 1 = -\frac{2}{3}.$ This answer might also be obtained if the student uses parentheses correctly but makes an algebraic mistake by taking $(-1)^2 = -1$ to get: $\frac{x^3}{3} - x^2\Big\vert_{-1}^{3} = \left(\frac{27}{3} - 9\right) - \left(-\frac{1}{3} - (-1)\right) = 0 + \left(\frac{1}{3} - 1\right) = -\frac{2}{3}.$
(B)	**This option is correct.** This question involves using the basic power rule for antidifferentiation and then correctly substituting the endpoints and evaluating: $\frac{x^3}{3} - x^2\Big\vert_{-1}^{3} = \left(\frac{27}{3} - 9\right) - \left(-\frac{1}{3} - 1\right) = \frac{4}{3}.$
(C)	This option is incorrect. If the process of antidifferentiation and differentiation are confused in evaluating the definite integral, the result is: $2x - 2\vert_{-1}^{3} = 8.$

(D)	This option is incorrect. A possible error in applying the power rule for antiderivatives is to not divide by the new exponent. This error would give: $x^3 - 2x^2 \Big\|_{-1}^{3} = 12.$

Question 2

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.1C Calculate derivatives.	**2.1C3:** Sums, differences, products, and quotients of functions can be differentiated using derivative rules.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 5:** Building notational fluency

(A)	This option is incorrect. Subtraction was used in the product rule instead of addition. $f'(x) = 2e^{2x}\left(x^3 + 1\right) - e^{2x}\left(3x^2\right)$ $f'(2) = 18e^4 - 12e^4 = 6e^4$
(B)	This option is incorrect. The correct product rule was used but the chain rule is ignored in the derivative of e^{2x}, thereby a factor of 2 was lost. $f'(x) = e^{2x}\left(x^3 + 1\right) + e^{2x}\left(3x^2\right)$ $f'(2) = 9e^4 + 12e^4 = 21e^4$
(C)	This option is incorrect. The chain rule was correctly used but the product rule is incorrectly stated as being $\frac{d}{dx}(f(x)g(x)) = f'(x)g'(x).$ $f'(x) = 2e^{2x}\left(3x^2\right)$ $f'(2) = 2e^4\left(3 \cdot 2^2\right) = 24e^4$
(D)	**This option is correct.** Use a combination of the product rule and the chain rule to compute the derivative, then evaluate that derivative at $x = 2$. $f'(x) = 2e^{2x}\left(x^3 + 1\right) + e^{2x}\left(3x^2\right)$ $f'(2) = 18e^4 + 12e^4 = 30e^4$

Question 3

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.3B Solve problems involving the slope of a tangent line.	**2.3B2:** The tangent line is the graph of a locally linear approximation of the function near the point of tangency.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes

(A)	This option is incorrect. An incorrect equation was used, namely $y = f(a) + f'(a)x,$ in constructing the equation of a line given a slope and a point on the line. $f(2.1) \approx f(2) - \frac{1}{2}(2.1) = 4 - 1.05 = 2.95$
(B)	**This option is correct.** An equation of the tangent line at $x = a$ is $y = f(a) + f'(a)(x - a).$ In this question $a = 2$ and $f'(a) = -\frac{1}{2}.$ The value of y when $x = 2.1$ would be an approximation to $f(2.1).$ $f(2.1) \approx f(2) - \frac{1}{2}(2.1 - 2) = 4 - \frac{1}{2}(0.1) = 4 - 0.05 = 3.95$
(C)	This option is incorrect. An error was made in constructing the equation of a line given a slope and a point on the line, using $y = f(a) - f'(a)(x - a)$ for the equation of the tangent line. $f(2.1) \approx f(2) + \frac{1}{2}(2.1 - 2) = 4 + \frac{1}{2}(0.1) = 4.05$ The same answer could also be obtained with the correct tangent line equation by making a multiplication error or sign error in substituting the derivative value or performing the calculation.
(D)	This option is incorrect. This is a fundamental conceptual error in not recognizing the derivative as the slope of the tangent line. $f(2.1) \approx f(2) + (2.1 - 2) = 4 + 0.1 = 4.1$

Question 4

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.2A Use derivatives to analyze properties of a function.	**2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 2:** Connecting concepts

(A)	This option is incorrect. The error comes from believing that g is always increasing because the derivative is the sum of two terms and is thus always positive.
(B)	**This option is correct.** First find the critical values where the derivative is 0 or undefined. A relative extrema occurs at a critical value if the derivative changes sign there. $g'(x) = 4x^3 + 12x^2 = x^2(4x + 12)$ $x^2(4x + 12) = 0$ at $x = 0$ or $x = -3$ There are two zeros, but $g'(x)$ changes sign for $x = -3$ only. The sign of $g'(x)$ at $x = 0$ does not change because of the x^2 factor.
(C)	This option is incorrect. The number of critical values has been confused with the number of relative extrema. There are two zeros for $g'(x) = 0,$ but the derivative only changes sign at one of them.
(D)	This option is incorrect. The error is the result of assuming that a quartic polynomial always has 3 relative extrema or that a cubic polynomial always has 3 zeros.

Question 5

<table>
<tr><th colspan="2">Learning Objective</th><th>Essential Knowledge</th><th>Mathematical Practices for AP Calculus</th></tr>
<tr><td colspan="2">3.4B Apply definite integrals to problems involving the average value of a function.</td><td>3.4B1: The average value of a function f over an interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx.$</td><td>MPAC 1: Reasoning with definitions and theorems
MPAC 3: Implementing algebraic/ computational processes</td></tr>
<tr><td>(A)</td><td colspan="3">This option is incorrect. The average velocity has been confused with the average rate of change of velocity over the interval:
$\frac{v(3)-v(1)}{3-1} = \frac{-7-1}{2} = -4.$</td></tr>
<tr><td>(B)</td><td colspan="3">This option is incorrect. The average velocity is not the arithmetic mean of the velocity values at the endpoints:
$\frac{v(3)+v(1)}{2} = \frac{-7+1}{2} = -3.$</td></tr>
<tr><td>(C)</td><td colspan="3">This option is correct. The average value of $v(t)$ over the interval $[1, 3]$ is computed with a definite integral:
$\frac{1}{3-1}\int_1^3 v(t)\,dt = \frac{1}{3-1}\int_1^3 \left(2-t^2\right)dt = \frac{1}{2}\left(2t - \frac{1}{3}t^3\Big|_1^3\right)$
$= \frac{1}{2}\left((6-9) - \left(2-\frac{1}{3}\right)\right) = -\frac{7}{3}.$</td></tr>
<tr><td>(D)</td><td colspan="3">This option is incorrect. Speed and velocity have been confused and it was assumed that the average velocity must be positive:
$\left|\frac{1}{3-1}\int_1^3 \left(2-t^2\right)dt\right| = \frac{7}{3}.$</td></tr>
</table>

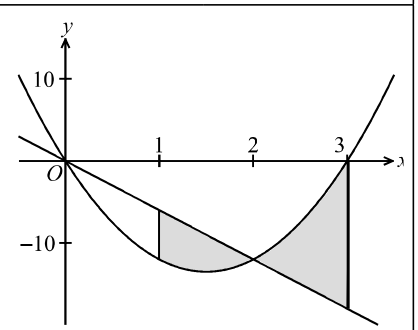
Question 6

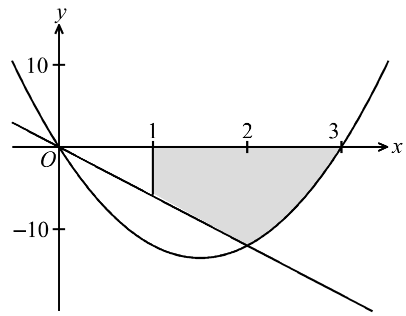
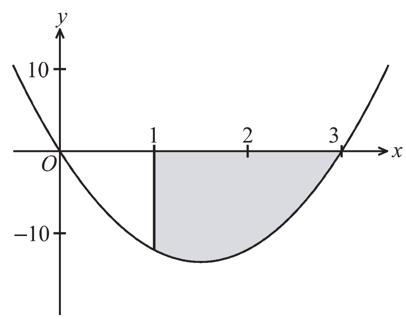
Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.2B Approximate a definite integral.	**3.2B2:** Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 3:** Implementing algebraic/ computational processes

(A)	**This option is correct.** The trapezoidal sum is the average of the left and right Riemann sums. The three intervals are of length 2, 5, and 2, respectively. Taking the average of the left and right endpoint values on each interval and multiplying by the length of the interval gives the following trapezoidal sum: $\frac{2}{2}(15+9)+\frac{5}{2}(9+5)+\frac{2}{2}(5+4)=68.$
(B)	This option is incorrect. Using the trapezoidal rule is not appropriate because the intervals are of different lengths. Ignoring the values of t and believing that $\Delta t=\frac{9-0}{3}=3$ as if the intervals were of equal length in the table would yield the following computation using the trapezoidal rule: $\frac{3}{2}[15+2(9)+2(5)+4]=70.5.$
(C)	This option is incorrect. A trapezoidal sum has been confused with a left Riemann sum: $2\cdot 15+5\cdot 9+2\cdot 5=85.$
(D)	This option is incorrect. The error is due to taking just the sum of the left and right Riemann sums rather than the average, thus not dividing each width by 2: $2(15+9)+5(9+5)+2(5+4)=136.$

Question 7

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.4D Apply definite integrals to problems involving area, volume, (BC) and length of a curve.	**3.4D1:** Areas of certain regions in the plane can be calculated with definite integrals. (BC) Areas bounded by polar curves can be calculated with definite integrals.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes
3.3B(b) Evaluate definite integrals.	**3.3B2:** If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$.	

(A)	This option is incorrect. If a sketch is not made or if it is not checked first whether the two graphs intersect on the interval $1 \le x \le 3$, it is likely that the calculation for area will be done with just one integral: $\int_1^3 (f(x) - g(x))\,dx = \int_1^3 \left(6x^2 - 12x\right) dx$ $= \left(2x^3 - 6x^2\right)\Big\|_1^3 = (54 - 54) - (2 - 6).$ If the integration had been done as $\int_1^3 (g(x) - f(x))\,dx,$ the answer would be $-4.$ But since area must be positive, the absolute value would give the same answer as the first integral.
(B)	**This option is correct.** It is helpful to sketch the graphs of the two functions and to check if the two graphs cross inside the given interval. The first graph is a concave up parabola with zeros at $x = 0$ and $x = 3$, and the second graph is a line passing through the origin. The sketch indicates that there are two regions between the curves on the interval $1 \le x \le 3$. The line intersects the parabola where $6x^2 - 18x = -6x,$ or $0 = 6x^2 - 12x = 6x(x - 2);$ therefore the top and bottom curves of the regions switch roles at $x = 2.$ Different integrals should be used to find the total area of the regions, one for the interval $1 \le x \le 2$ and one for the interval $2 \le x \le 3$. Let $f(x) = 6x^2 - 18x$ and $g(x) = -6x$. The total area is:  $\int_1^3 \|f(x) - g(x)\|\,dx = \int_1^2 (g(x) - f(x))\,dx + \int_2^3 (f(x) - g(x))\,dx$ $= \int_1^2 \left(12x - 6x^2\right) dx + \int_2^3 \left(6x^2 - 12x\right) dx = \left(6x^2 - 2x^3\right)\Big\|_1^2 + \left(2x^3 - 6x^2\right)\Big\|_2^3$ $= ((24 - 16) - (6 - 2)) + ((54 - 54) - (16 - 24)) = 4 + 8 = 12.$

(C)	This option is incorrect. The error comes from finding the area bounded by the curves and the x-axis between $x = 1$ and $x = 3$. Since the region is below the axis, the area is the opposite of the integral value: $\left(-\int_1^2 g(x)\,dx\right) + \left(-\int_2^3 f(x)\,dx\right)$.	
(D)	This option is incorrect. The error comes from finding the area bounded only by the parabola and x-axis between $x = 1$ and $x = 3$. Because the region is below the axis, the area is the opposite of the integral value: $-\int_1^3 f(x)\,dx$.	

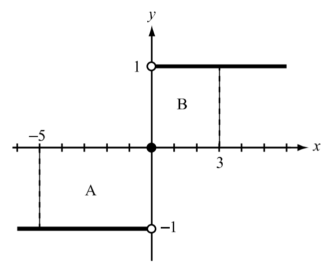
Question 8

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.3B Solve problems involving the slope of a tangent line.	**2.3B1:** The derivative at a point is the slope of the line tangent to a graph at that point on the graph.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 2:** Connecting concepts

(A)	This option is incorrect. The slope of the tangent line has been incorrectly set equal to $g(-1)$ instead of $g'(-1)$: $g(-1) = (-1)^2 + b(-1) = 2$ $\Rightarrow\ b = -1.$
(B)	This option is incorrect. Only the slope of the line was calculated through the two given points, but that was not connected to the calculation to find the value of b.
(C)	This option is incorrect. The slope of the line through $(0, -2)$ and $(3, 4)$ was mistakenly found using the wrong difference quotient, $\frac{\Delta x}{\Delta y} = \frac{1}{2}.$ $g'(x) = 2x + b$ $g'(-1) = 2(-1) + b = \frac{1}{2} \Rightarrow b = \frac{5}{2}$
(D)	**This option is correct.** Parallel lines have the same slope. The slope of the tangent line at $x = -1$ is therefore equal to the slope of the line through $(0, -2)$ and $(3, 4),$ which is $\frac{4-(-2)}{3-0} = 2.$ Therefore find b so that $g'(-1) = 2.$ $g'(x) = 2x + b$ $g'(-1) = 2(-1) + b = 2 \Rightarrow b = 4$

Question 9

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.2C Calculate a definite integral using areas and properties of definite integrals.	**3.2C3:** The definition of the definite integral may be extended to functions with removable or jump discontinuities.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 5:** Building notational fluency

(A)	**This option is correct.** This function has a jump discontinuity at $x = 0$. $f(x) = -1$ for $x < 0$ and $f(x) = 1$ for $x > 0$. The graph of f is shown to the right along with the regions A and B between the graph of f and the x-axis on the intervals $[-5, 0]$ and $[0, 3]$, respectively.  Computing the definite integral in terms of areas and taking into account which region is below the axis and which is above, we get: $\int_{-5}^{3} f(x)\,dx = -\text{area}(A) + \text{area}(B) = -5 + 3 = -2.$ The definite integral over the interval $[-5, 3]$ can also be written as the sum of the definite integrals over $[-5, 0]$ and $[0, 3]$, giving: $\int_{-5}^{3} f(x)\,dx = \int_{-5}^{0} -1\,dx + \int_{0}^{3} 1\,dx = -5 + 3 = -2.$
(B)	This option is incorrect. Sign errors were made in the definition of f so that the graph of f is reflected across the x-axis. This then yields: $\int_{-5}^{3} f(x)\,dx = \text{area}(A) - \text{area}(B) = 5 - 3 = 2$ or $\int_{-5}^{3} f(x)\,dx = \int_{-5}^{0} 1\,dx + \int_{0}^{3} -1\,dx = 5 - 3 = 2.$
(C)	This option is incorrect. Rather than taking into account the signed areas, this is the total area: $\int_{-5}^{3} f(x)\,dx = \text{area}(A) + \text{area}(B) = 5 + 3 = 8$ or $\int_{-5}^{3} f(x)\,dx = \left\lvert \int_{-5}^{0} -1\,dx \right\rvert + \left\lvert \int_{0}^{3} 1\,dx \right\rvert = 5 + 3 = 8.$
(D)	This option is incorrect. The source of the error may be not recognizing that the definition of the definite integral can be extended to functions with removable or jump discontinuities.

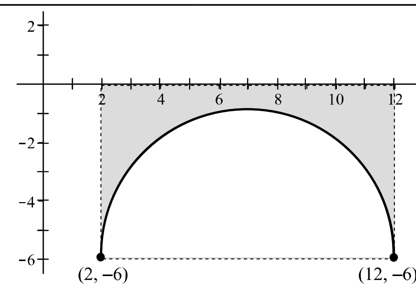
Question 10

Learning Objective	**Essential Knowledge**	**Mathematical Practices for AP Calculus**
3.3B(b) Evaluate definite integrals.	**3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, (BC) integration by parts, and nonrepeating linear partial fractions.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 5:** Building notational fluency

(A)	This option is incorrect. The technique of substitution of variables was not applied correctly because only the substitution $u = 2x - 1$ was made. There is no substitution for dx or for the limits of integration.
(B)	This option is incorrect. The technique of substitution of variables was not applied correctly because only the substitutions for $2x - 1$ and dx were made, but not for the limits of integration.
(C)	This option is incorrect. The technique of substitution of variables was not applied correctly because only the substitutions for $2x - 1$ and the limits of integration were made, but not for dx.
(D)	**This option is correct.** Starting with the substitution $u = 2x - 1$, $u = 2x - 1 \Rightarrow du = 2dx \Rightarrow dx = \frac{1}{2}du.$ Also change the limits of integration for x to limits of integration for u: $x = 2 \Rightarrow u = 2 \cdot 2 - 1 = 3$ $x = 3 \Rightarrow u = 2 \cdot 3 - 1 = 5.$ Substituting for $2x - 1,$ for $dx,$ and for the limits of integration gives: $\int_2^3 g(2x-1)\,dx = \int_3^5 g(u) \cdot \frac{1}{2}\,du = \frac{1}{2}\int_3^5 g(u)\,du.$

Question 11

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.2C Calculate a definite integral using areas and properties of definite integrals.	**3.2C1:** In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 4:** Connecting multiple representations

(A)	This option is incorrect. Only the area of the semicircle was considered. The opposite of that area was then taken, since the region is below the axis.
(B)	This option is incorrect. Only the area of the semicircle was considered, ignoring both the area of the rectangle and the fact that the region is below the axis.
(C)	**This option is correct.** The definite integral can be evaluated by using geometry and the connection of the definite integral with areas. In particular, the value of the integral here is the opposite of the area of the region between the semicircle and the x-axis, i.e. the area of the rectangle with vertices at $(2, 0)$, $(12, 0)$, $(2, -6)$, and $(12, -6)$ minus the area of the semicircle.  The area of the rectangle is $6 \cdot 10 = 60$. The area of the semicircle of radius 5 is $\frac{1}{2}\pi(5)^2 = \frac{25\pi}{2}$. The definite integral is therefore equal to $-\left(60 - \frac{25\pi}{2}\right)$.
(D)	This option is incorrect. Only the area of the rectangle minus the area of the semicircle was computed; the need to take the opposite of this value, since the region is below the axis, was not recognized.

Question 12

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.5A Analyze differential equations to obtain general and specific solutions.	**3.5A1:** Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line, exponential growth and decay, (BC) and logistic growth.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 2:** Connecting concepts

(A)	This option is incorrect. This is the velocity at $t = 2,$ not the position: $v(t) = 3t^2 + 10 \Rightarrow v(2) = 22.$ The velocity at $t = 2$ can also be found directly using the Fundamental Theorem of Calculus: $v(2) = v(0) + \int_0^2 v'(t)\,dt = 10 + \int_0^2 a(t)\,dt = 10 + \int_0^2 6t\,dt = 22.$
(B)	This option is incorrect. The assumption was made that the velocity is constant and the acceleration was ignored: $p(2) = 7 + 2 \cdot v(0) = 7 + 2 \cdot 10 = 27.$
(C)	This option is incorrect. This is the total change in position without accounting for the initial position. The calculation uses the correct velocity function but takes $p(t) = t^3 + 10t.$
(D)	**This option is correct.** Velocity is the antiderivative of acceleration and position is the antiderivative of velocity. In each case, the object's velocity and position at $t = 0$ can be used to find the appropriate "+ C" after each antidifferentiation. The last step is to then evaluate the position function at $t = 2.$ $a(t) = 6t$ $v(t) = 3t^2 + C_1$ $v(0) = 3(0)^2 + C_1 = 10 \Rightarrow C_1 = 10$ $v(t) = 3t^2 + 10$ $p(t) = t^3 + 10t + C_2$ $p(0) = (0)^3 + 10(0) + C_2 = 7 \Rightarrow C_2 = 7$ $p(t) = t^3 + 10t + 7$ $p(2) = 2^3 + 10(2) + 7 = 35$

Question 13

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.1D Determine higher order derivatives.	**2.1D1:** Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher order derivatives of f.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 5:** Building notational fluency

(A)	**This option is correct.** This question requires repeated differentiation using knowledge of the derivatives of sine and cosine, the derivative of the natural logarithm, and the power rule for derivatives, as well as recognizing the need to use the chain rule for the first derivative. $\frac{dy}{dx} = -\sin x - \frac{2}{2x} = -\sin x - \frac{1}{x}$ $\frac{d^2y}{dx^2} = -\cos x + \frac{1}{x^2}$ $\frac{d^3y}{dx^3} = \sin x - \frac{2}{x^3}$
(B)	This option is incorrect. A consistent sign error was made with the derivatives of the two trigonometric functions by using $\frac{d}{dx}(\sin x) = -\cos x$ and $\frac{d}{dx}(\cos x) = \sin x$. $\frac{dy}{dx} = \sin x - \frac{2}{2x} = \sin x - \frac{1}{x}$ $\frac{d^2y}{dx^2} = -\cos x + \frac{1}{x^2}$ $\frac{d^3y}{dx^3} = -\sin x - \frac{2}{x^3}$

(C)	This option is incorrect. The only error was not using the chain rule when taking the first derivative. No chain rule is needed after that: $\frac{dy}{dx} = -\sin x - \frac{1}{2x}$ $\frac{d^2y}{dx^2} = -\cos x + \frac{1}{2x^2}$ $\frac{d^3y}{dx^3} = \sin x - \frac{2}{2x^3} = \sin x - \frac{1}{x^3}.$
(D)	This option is incorrect. A chain rule error was made on the first derivative, and there are consistent sign errors with the derivatives of the trigonometric functions. $\frac{dy}{dx} = \sin x - \frac{1}{2x}$ $\frac{d^2y}{dx^2} = -\cos x + \frac{1}{2x^2}$ $\frac{d^3y}{dx^3} = -\sin x - \frac{2}{2x^3} = -\sin x - \frac{1}{x^3}$

Question 14

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.3A Analyze functions defined by an integral.	**3.3A3:** Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x) = \int_a^x f(t)\,dt$.	**MPAC 4:** Connecting multiple representations **MPAC 1:** Reasoning with definitions and theorems
3.2C Calculate a definite integral using areas and properties of definite integrals.	**3.2C1:** In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.	

(A)	This option is incorrect. The Fundamental Theorem of Calculus was applied to calculate values of $g(x)$, but all the areas were added to the value $g(2)$. Therefore the conclusion was that $g(x) \geq 5$ for all values of x.
(B)	This option is incorrect. It comes from confusing the graph of g with the graph of f and finding the number of times that $f(x) = 3$, which happens only between $x = 5$ and $x = 6$.
(C)	This option is incorrect. The Fundamental Theorem of Calculus was applied correctly to find values of $g(x)$ for $x > 2$, thus finding the solutions $c = 4$ and $5 < c < 6$. However, the integral $\int_2^0 f(x)\,dx$ might have been mishandled, thus missing the solution at $c = 0$.
(D)	**This option is correct.** By the Fundamental Theorem of Calculus and using area and geometry to calculate the definite integral, $g(0) = g(2) + \int_2^0 g'(x)\,dx = 5 - \int_0^2 f(x)\,dx = 5 - 2 = 3.$ So $c = 0$ is one solution to $g(x) = 3$. Since $g' = f$, the graph of f indicates that g is increasing from $x = 0$ to $x = 2$ since f is positive there. From $x = 2$ to $x = 4$ the function g decreases by the same amount that it increased from $x = 0$ to $x = 2$ by the symmetry of the regions. $g(4) = 5 + \int_2^4 f(x)\,dx = 5 + (-2) = 3$ This gives a second solution at $c = 4$. According to the graph of the derivative f, the function g continues to decrease until $x = 5$ and then begins increasing again. $g(5) = 5 + \int_2^5 f(x)\,dx = 5 + (-3) = 2$ $g(6) = 2 + \int_5^6 f(x)\,dx = 2 + 2 = 4$ Since g is continuous, the Intermediate Value Theorem guarantees a third solution somewhere on the interval $(5, 6)$. There are no other solutions possible. (The third solution is at $c = 5.5$.)

Question 15

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
1.2A Analyze functions for intervals of continuity or points of discontinuity.	**1.2A1:** A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim_{x \to c} f(x)$ exists, and $\lim_{x \to c} f(x) = f(c)$.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 5:** Building notational fluency

(A)	This option is incorrect. Rather than setting as equal the left- and right-sided limits at $x = 1$, the derivatives of the two pieces were set equal at $x = 1$. $\frac{d}{dx}(6 + cx) = \frac{d}{dx}(9 + 2\ln x)$ $c = \frac{2}{x} \Rightarrow c = 2$
(B)	**This option is correct.** Since the function f is continuous at $x = 1$, then $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$. Evaluate these two limits and set them equal, then solve for c. $\lim_{x \to 1^-} f(x) = 6 + c \cdot 1 = 6 + c$ $\lim_{x \to 1^+} f(x) = 9 + 2\ln 1 = 9$ $6 + c = 9 \Rightarrow c = 3$
(C)	This option is incorrect. Rather than setting as equal the left- and right-sided limits at $x = 1$, the derivatives of the two pieces were set equal at $x = 1$, but a calculus error was made with respect to the derivative of a constant. $\frac{d}{dx}(6 + cx) = \frac{d}{dx}(9 + 2\ln x)$ $6 + c = 9 + \frac{2}{x}$ $6 + c = 9 + 2 \Rightarrow c = 5$ OR The left- and right-sided limits were set equal at $x = 1$, but an algebraic error was made in evaluating $\ln 1 = 1$: $6 + c(1) = 9 + 2(1) = 11 \Rightarrow c = 5.$
(D)	This option is incorrect. The value of the function at $x = 1$ was confused with c so that $c = f(1) = 9 + 2\ln 1 = 9$.

Question 16

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.3F Estimate solutions to differential equations.	**2.3F1:** Slope fields provide visual clues to the behavior of solutions to first order differential equations.	**MPAC 4:** Connecting multiple representations **MPAC 2:** Connecting concepts

(A)	This option is incorrect. For this differential equation, the slopes of the line segments in Quadrant II must be positive since $y > 0$ and $x^2 > 0$ in that quadrant. In this slope field that does not happen, as can be observed with the segments near the bottom left of Quadrant II. In addition, all line segments along the x-axis should have positive slopes. This is not the case here. This slope field might be chosen if the squared term is not accounted for. [This is the slope field for $\frac{dy}{dx} = x + y$.]
(B)	This option is incorrect. For this differential equation, the slopes of the line segments in Quadrant I must be positive since $y > 0$ and $x^2 > 0$ in that quadrant. In this slope field, however, all the line segments in Quadrant I have negative slopes. In addition, all line segments along the x-axis should have positive slopes. This is not the case here. This slope field might be chosen if one considers $\frac{dy}{dx} = 0$ and thinks of the differential equation as relating to the parabola $y = -x^2$. [This is the slope field for $\frac{dy}{dx} = -1.8x$.]
(C)	This option is incorrect. For this differential equation, the slopes of the line segments in Quadrant II must be positive since $y > 0$ and $x^2 > 0$ in that quadrant. In this slope field, however, that does not happen as can be observed with the segments near the bottom left of Quadrant II. In addition, all line segments along the x-axis should have positive slopes. This is not the case here. This option might be chosen if the x and y variables are confused and one looks for the slope field for $\frac{dy}{dx} = x + y^2$.
(D)	**This option is correct.** The line segments in the slope field have slopes given by $\frac{dy}{dx} = x^2 + y$ at the point (x, y). In Quadrants I and II, all slopes must be positive or zero since $y > 0$ in those quadrants and $x^2 \geq 0$. This is the only option in which that condition is true.

Question 17

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.3E Verify solutions to differential equations.	**2.3E2:** Derivatives can be used to verify that a function is a solution to a given differential equation.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes

(A)	**This option is correct.** One way to verify that a function is a solution to a differential equation is to check that the function and its derivatives satisfy the differential equation. The differential equations in this problem involve both y' and y''. The correct derivatives must be computed and the algebra correctly done to verify that the differential equation is satisfied. $y' = 3e^{3x} - 5$ $y'' = 9e^{3x}$ $y'' - 3y' - 15 = 9e^{3x} - 3\left(3e^{3x} - 5\right) - 15$ $= 9e^{3x} - 9e^{3x} + 15 - 15 = 0$
(B)	This option is incorrect. The correct derivatives were found but this differential equation may appear to be satisfied if the subtraction in the second term is not distributed correctly across the two terms inside the parentheses: $y'' - 3y' + 15 = 9e^{3x} - 3\left(3e^{3x} - 5\right) + 15 = 9e^{3x} - 9e^{3x} - 15 + 15 = 0.$
(C)	This option is incorrect. This differential equation will appear to be satisfied if the chain rule is not used in taking the derivative of the exponential: $y' = e^{3x} - 5$ $y'' = e^{3x}$ (error) $y'' - y' - 5 = e^{3x} - \left(e^{3x} - 5\right) - 5 = e^{3x} - e^{3x} + 5 - 5 = 0.$
(D)	This option is incorrect. This differential equation will appear to be satisfied if the chain rule is not used in taking the derivative of the exponential and the subtraction in the second term is not distributed correctly across the two terms inside the parentheses: $y' = e^{3x} - 5$ $y'' = e^{3x}$ (error) $y'' - y' + 5 = e^{3x} - \left(e^{3x} - 5\right) + 5 = e^{3x} - e^{3x} - 5 + 5 = 0.$

Question 18

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.1C Calculate derivatives.	**2.1C2:** Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 5:** Building notational fluency

(A)	This option is incorrect. The incorrect derivative, $\frac{d}{dx}\sin^{-1}x = \cos^{-1}x$, was used but the inverse cosine function was correctly evaluated to get $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$.
(B)	This option is incorrect. The incorrect derivative, $\frac{d}{dx}\sin^{-1}x = \cos^{-1}x$, was used and the inverse cosine function was incorrectly evaluated as $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$.
(C)	This option is incorrect. The derivative of the inverse sine function was confused with the derivative of the inverse tangent function, and therefore $\frac{1}{1+x^2}$ was used for the derivative: $\frac{1}{1+\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{1+\frac{3}{4}} = \frac{4}{7}$.
(D)	**This option is correct.** The derivative of the inverse sine function $\sin^{-1}x$ is $\frac{1}{\sqrt{1-x^2}}$. Since $f(x) = \sin^{-1}x$, $f'\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}} = \frac{1}{\sqrt{\frac{1}{4}}} = 2$.

Question 19

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.1A Identify the derivative of a function as the limit of a difference quotient.	**2.1A2:** The instantaneous rate of change of a function at a point can be expressed by $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ or $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 5:** Building notational fluency

(A)	This option is incorrect. This could have been caused by observing that the numerator is zero at $x = e$ and then ignoring the denominator.
(B)	**This option is correct.** The limit of this difference quotient is of the form $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ where $f(x) = x^{20} - 3x$ and $a = e$. This is one way to express the derivative of f at $x = e$. $f'(x) = \frac{d}{dx}\left(x^{20} - 3x\right) = 20x^{19} - 3$ $\Rightarrow f'(e) = 20e^{19} - 3$
(C)	This option is incorrect. This is just the value of the function $x^{20} - 3x$ at $x = e$.
(D)	This option is incorrect. Observing that the denominator was zero at $x = e$ could have led to the assumption of unbounded behavior while the numerator was ignored. Or it was thought that the indeterminate form $\frac{0}{0}$ produced a nonexistent result, without recognizing the limit as involving the definition of the derivative.

Question 20

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.1C Calculate derivatives.	**2.1C5:** The chain rule is the basis for implicit differentiation.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 5:** Building notational fluency
2.1D Determine higher order derivatives.	**2.1D1:** Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher order derivatives of f.	

(A)	This option is incorrect. The chain rule may not have been used. $\frac{d^2y}{dx^2} = 3y^2 \Rightarrow \left.\frac{d^2y}{dx^2}\right\rvert_{(x,\, y)=(1,\, 2)} = 3(4) = 12$ The x and y values could also have been reversed when substituting into the second derivative. $\frac{d^2y}{dx^2} = 3y^2\frac{dy}{dx} = 3y^2\left(y^3 + 3\right)$ $\left.\frac{d^2y}{dx^2}\right\rvert_{(x,\, y)=(1,\, 2)} = 3(1)(4) = 12$
(B)	This option is incorrect. There was an error in the power rule when differentiating y^3 resulted in $3y$. $\frac{d^2y}{dx^2} = 3y\frac{dy}{dx} = 3y\left(y^3 + 3\right)$ $\left.\frac{d^2y}{dx^2}\right\rvert_{(x,\, y)=(1,\, 2)} = 3(2)\left(2^3 + 3\right) = 3(2)(11) = 66$
(C)	**This option is correct.** The answer is obtained by using the chain rule when doing implicit differentiation. The second derivative is found in terms of y and $\frac{dy}{dx}$, which can then be written completely in terms of y. The condition $f(1) = 2$ means that $y = 2$ when $x = 1$. $\frac{d^2y}{dx^2} = 3y^2\frac{dy}{dx} = 3y^2\left(y^3 + 3\right)$ $\left.\frac{d^2y}{dx^2}\right\rvert_{(x,\, y)=(1,\, 2)} = 3 \cdot 2^2\left(2^3 + 3\right) = 3(4)(11) = 132$

(D)	This option is incorrect. There was an error in differentiating a constant function and not getting 0 (i.e. not differentiating the 3). $\frac{d^2y}{dx^2} = \left(3y^2 + 3\right)\frac{dy}{dx} = \left(3y^2 + 3\right)\left(y^3 + 3\right)$ $\left.\frac{d^2y}{dx^2}\right\|_{(x,\ y)=(1,\ 2)} = (3 \cdot 4 + 3)(11) = 165$

Question 21

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.1C Calculate derivatives.	**2.1C4:** The chain rule provides a way to differentiate composite functions.	**MPAC 5:** Building notational fluency **MPAC 4:** Connecting multiple representations

(A)	This option is incorrect. The notation $f(g(x))$ was confused with the product $f(x)g(x)$ and the product rule was used to find the derivative: $h'(1) = f'(1)g(1) + g'(1)f(1)$ $h'(1) = (-3)(3) + (-2)(5) = -19.$
(B)	**This option is correct.** The solution is found by using the chain rule correctly and then using values of the functions and their derivatives from the table to evaluate the derivative at $x = 1$. $h'(1) = f'(g(1))g'(1)$ $h'(1) = f'(3)g'(1)$ $h'(1) = 7(-2) = -14$
(C)	This option is incorrect. The chain rule for composite functions was incompletely understood by thinking that $\frac{d}{dx}f(g(x)) = f'(g(x))$: $h'(1) = f'(g(1)) = 7.$
(D)	This option is incorrect. The chain rule for composite functions was misunderstood by thinking that $\frac{d}{dx}f(g(x)) = f'(g'(x))$: $h'(1) = f'(g'(1))$ $h'(1) = f'(-2)$ $h'(1) = 9.$

Question 22

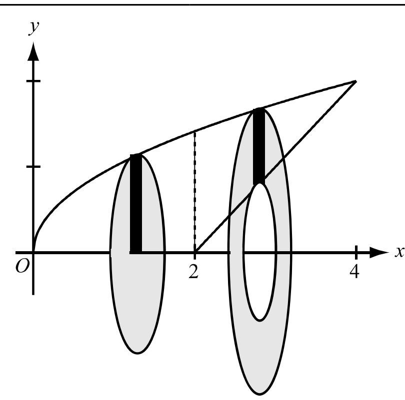
Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.5A Analyze differential equations to obtain general and specific solutions.	**3.5A2:** Some differential equations can be solved by separation of variables.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 5:** Building notational fluency

(A)	This option is incorrect. The arbitrary constant must be included when doing the antidifferentiation, not after solving for y (sometimes referred to as "The *Late C*"). This is the result if the constant is introduced at the wrong time. $\frac{dy}{dx} = \frac{x+1}{y} \Rightarrow y\,dy = (x+1)\,dx$ $\int y\,dy = \int (x+1)\,dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + x \Rightarrow y^2 = x^2 + 2x$ $y = \pm\sqrt{x^2 + 2x} + C$ $-2 = 0 + C$ $y = -\sqrt{x^2 + 2x} - 2$
(B)	This option is incorrect. The arbitrary constant was introduced after solving for y (see the steps in option (A)), and the positive square root was selected.
(C)	**This option is correct.** This question involves solving a differential equation by separation of variables and using the initial condition to determine the appropriate value for the arbitrary constant. The solution leads to y^2. Since the initial value of y is negative, in solving for y the negative square root is used. $\frac{dy}{dx} = \frac{x+1}{y} \Rightarrow y\,dy = (x+1)\,dx$ $\int y\,dy = \int (x+1)\,dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + x + C$ $\frac{(-2)^2}{2} = 0 + 0 + C \Rightarrow 2 = C$ $\frac{y^2}{2} = \frac{x^2}{2} + x + 2 \Rightarrow y^2 = x^2 + 2x + 4$ $y = -\sqrt{x^2 + 2x + 4}$
(D)	This option is incorrect. All the calculus steps were done correctly as in the calculations for option (C), but the positive square root was used at the last step without consideration of the sign of the initial y value.

Question 23

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.4D Apply definite integrals to problems involving area, volume, (BC) and length of a curve.	**3.4D2:** Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.	**MPAC 4:** Connecting multiple representations **MPAC 5:** Building notational fluency

<table>
<tr><td>(A)</td><td>This option is incorrect. This expression uses the entire region between the curves from $x = 0$ to $x = 4,$ including the triangle below the x-axis from $x = 0$ to $x = 2$ that is not part of region R. It treats every rotated slice on the entire interval as if forming a washer.</td></tr>
<tr><td>(B)</td><td>This option is incorrect. This expression uses the entire region between the curves from $x = 0$ to $x = 4,$ including the triangle below the x-axis from $x = 0$ to $x = 2$ that is not part of region R. It treats every rotated slice on the entire interval as forming a washer, and also makes the mistake of using $\pi\left(r_2 - r_1\right)^2$ instead of $\pi\left(r_2{}^2 - r_1{}^2\right)$ for the area of a washer with inner radius r_1 and outer radius r_2.</td></tr>
<tr><td>(C)</td><td>This option is correct. The region R must be split into two parts at $x = 2$ when considering its rotation around the x-axis. For the left part, rotating a typical slice around the x-axis will form a disc of radius $r = \sqrt{x}$. For the right part, rotating a typical slice around the x-axis will form a washer with inner radius $r_1 = x - 2$ and outer radius $r_2 = \sqrt{x}$. The combined volume will be given by the integral expression:



$$\pi\int_0^2 \left(\sqrt{x}\right)^2\, dx + \pi\int_2^4\left(\left(\sqrt{x}\right)^2 - (x-2)^2\right) dx$$
$$= \pi\int_0^2 x\, dx + \pi\int_2^4\left(x - (x-2)^2\right) dx.$$</td></tr>
<tr><td>(D)</td><td>This option is incorrect. The region was correctly split into two parts at $x = 2,$ but the second integral used $\pi\left(r_2 - r_1\right)^2$ instead of $\pi\left(r_2{}^2 - r_1{}^2\right)$ for the area of a washer with inner radius r_1 and outer radius r_2.</td></tr>
</table>

Question 24

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
1.1C Determine limits of functions.	**1.1C3:** Limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ may be evaluated using L'Hospital's Rule.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 5:** Building notational fluency

(A)	This option is incorrect. It may have been observed that the numerator is zero at $x = 3$ and the denominator was then ignored.
(B)	This option is incorrect. The indeterminate form $\frac{0}{0}$ was recognized and an attempt was made to use L'Hospital's Rule, but an error was made in the derivative of the denominator: $\lim_{x \to 3} \frac{\tan(x-3)}{3e^{x-3} - x} = \lim_{x \to 3} \frac{\sec^2(x-3)}{3e^{x-3}} = \frac{1}{3}.$
(C)	**This option is correct.** Substituting $x = 3$ into the fraction yields the indeterminate form $\frac{0}{0}$. Therefore L'Hospital's Rule can be used to find the limit: $\lim_{x \to 3} \frac{\tan(x-3)}{3e^{x-3} - x} = \lim_{x \to 3} \frac{\sec^2(x-3)}{3e^{x-3} - 1} = \frac{1}{2}.$
(D)	This option is incorrect. The observation that the denominator is zero at $x = 3$ led to the assumption of unbounded behavior, while the numerator was ignored.

Question 25

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.4A Apply the Mean Value Theorem to describe the behavior of a function over an interval.	**2.4A1:** If a function f is continuous over the interval $[a, b]$ and differentiable over the interval (a, b), the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 3:** Implementing algebraic/computational processes

(A)	This option is incorrect. The error comes from thinking that the Mean Value Theorem guarantees a value of x in the open interval where the instantaneous rate of change of f is equal to the total change in f instead of the average rate of change over the interval. $f'(x) = f(3) - f(1) = 11 - 9 = 2$ $\frac{3x^2 - 6}{x^2} = 2 \Rightarrow 3x^2 - 6 = 2x^2$ $x^2 = 6$ $x = \sqrt{6}$
(B)	**This option is correct.** Because f is differentiable for $x > 0$, it is continuous on the closed interval $[1, 3]$ and differentiable on the open interval $(1, 3)$. Therefore the Mean Value Theorem guarantees that there is a number x with $1 < x < 3$ satisfying: $f'(x) = \frac{f(3) - f(1)}{3 - 1} = \frac{11 - 9}{3 - 1} = 1.$ Solve this equation for the value of x in the interval $1 < x < 3$. $\frac{3x^2 - 6}{x^2} = 1 \Rightarrow 3x^2 - 6 = x^2$ $2x^2 = 6$ $x = \sqrt{3}$
(C)	This option is incorrect. This is the x-value on the interval $1 < x < 3$ where $f'(x) = 0$. The Mean Value Theorem may have been confused with the Extreme Value Theorem.
(D)	This option is incorrect. This error comes from thinking that the Mean Value Theorem guarantees a value of x in the interval that is equal to the average rate of change over the interval: $x = \frac{f(3) - f(1)}{3 - 1} = \frac{11 - 9}{3 - 1} = 1.$

Question 26

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.3B(b) Evaluate definite integrals.	**3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, (BC) integration by parts, and nonrepeating linear partial fractions. **3.3B2:** If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 5:** Building notational fluency

(A) **This option is correct.** Before doing the integration, the integrand can be simplified using long division.

$$\begin{array}{r|l} & x - 3 \\ x + 2 & x^2 - x - 5 \\ & x^2 + 2x \\ \hline & -3x - 5 \\ & -3x - 6 \\ \hline & 1 \end{array}$$

Then:

$$\int_1^2 \frac{x^2 - x - 5}{x + 2}\,dx = \int_1^2 \left(x - 3 + \frac{1}{x + 2}\right) dx$$

$$= \frac{x^2}{2} - 3x + \ln(x + 2)\Big|_1^2 = (-4 + \ln 4) - \left(-\frac{5}{2} + \ln 3\right) = -\frac{3}{2} + \ln\frac{4}{3}.$$

(B) This option is incorrect. The error comes from believing that the definite integral $\int_1^2 \frac{f(x)}{g(x)}\,dx$ is equal to $\int_1^2 f(x)\,dx$ divided by $\int_1^2 g(x)\,dx$.

$$\frac{\frac{1}{3}x^3 - \frac{1}{2}x^2 - 5x\Big|_1^2}{\frac{1}{2}x^2 + 2x\Big|_1^2} = \frac{-\frac{25}{6}}{\frac{7}{2}} = -\frac{25}{21}$$

(C)	This option is incorrect. An algebraic error was made in the long division where terms other than the leading ones were added instead of being subtracted. $$\begin{array}{r} x+1 \\ x+2\overline{)x^2-x-5} \\ x^2+2x \\ \hline x-5 \\ x+2 \\ \hline -3 \end{array}$$ This would lead to: $$\int_1^2 \frac{x^2-x-5}{x+2}\,dx = \int_1^2 \left(x+1-\frac{3}{x+2}\right) dx$$ $$= \frac{x^2}{2}+x-3\ln(x+2)\Big\rvert_1^2 = (4-3\ln 4)-\left(\frac{3}{2}-3\ln 3\right) = \frac{5}{2}+3\ln\frac{3}{4}.$$
(D)	This option is incorrect. The antiderivative of $\frac{f(x)}{g(x)}$ is not the antiderivative of $f(x)$ divided by the antiderivative of $g(x)$. $$\left(\frac{\frac{1}{3}x^3-\frac{1}{2}x^2-5x}{\frac{1}{2}x^2+2x}\right)\Bigg\rvert_1^2 = -\frac{14}{9}+\frac{31}{15}=\frac{23}{45}$$

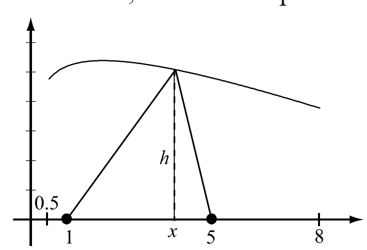
Question 27

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.3C Solve problems involving related rates, optimization, rectilinear motion, (BC) and planar motion.	**2.3C2:** The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 2:** Connecting concepts

(A)	**This option is correct.** The goal in this related rates problem is to find the value of $\frac{dA}{dt}$ at the instant when $h = 3$ and $\frac{dh}{dt} = -\frac{1}{2}$ (the derivative is negative since the depth of the water is decreasing). Using the chain rule gives: $\frac{dA}{dt} = \frac{dA}{dh} \cdot \frac{dh}{dt} = (12\pi - 2\pi h)\frac{dh}{dt}$ $\left.\frac{dA}{dt}\right\rvert_{h=3} = (12\pi - 6\pi)\left(-\frac{1}{2}\right) = -3\pi.$ The area is therefore decreasing at the rate of 3π square meters per minute.
(B)	This option is incorrect. A common mistake in related rates problems is to not use the chain rule, thereby having no $\frac{dh}{dt}$ term to include in the computation: $\frac{dA}{dt} = (12\pi - 2\pi h)$ $\left.\frac{dA}{dt}\right\rvert_{h=3} = (12\pi - 6\pi) = 6\pi.$
(C)	This option is incorrect. Each term in A was differentiated individually, but the chain rule should have been used with the first term because there is no longer an h variable present. The fact that the depth of the water is decreasing also was not taken into account. $\frac{dA}{dt} = 12\pi - 2\pi h\frac{dh}{dt}$ $\left.\frac{dA}{dt}\right\rvert_{h=3} = 12\pi - 2\pi(3)\left(\frac{1}{2}\right) = 9\pi$
(D)	This option is incorrect. This is the value of the area when $h = 3$, not the rate of change of the area.

Question 28

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.3C Solve problems involving related rates, optimization, rectilinear motion, (BC) and planar motion.	**2.3C3**: The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 2:** Connecting concepts

(A)	This option is incorrect. This is the area of the triangle with the third vertex at the endpoint for $x = \frac{1}{2}$: $A = \frac{1}{2}bh = \frac{1}{2}(4)\left(\ln(2x) - \frac{1}{2}x + 5\right) = 2\ln(2x) - x + 10.$ For $x = \frac{1}{2},\ A = 2\ln\left(2 \cdot \frac{1}{2}\right) - \frac{1}{2} + 10 = 0 - \frac{1}{2} + 10 = \frac{19}{2}.$
(B)	This option is incorrect. The chain rule was not used in calculating the derivative of the area function. $A = \frac{1}{2}bh = \frac{1}{2}(4)\left(\ln(2x) - \frac{1}{2}x + 5\right) = 2\ln(2x) - x + 10$ $\frac{dA}{dx} = 2\left(\frac{1}{2x}\right) - 1 = \frac{1}{x} - 1$ $\frac{dA}{dx} = \frac{1}{x} - 1 = 0 \Rightarrow x = 1$ $A = 2\ln(2 \cdot 1) - 1 + 10 = 2\ln 2 + 9$
(C)	**This option is correct.** The first step is to express the area of the triangle in terms of the variable x. To find the maximum area, the next step is to find the critical value for the area function. $A = \frac{1}{2}bh$ $= \frac{1}{2}(4)\left(\ln(2x) - \frac{1}{2}x + 5\right)$ $= 2\ln(2x) - x + 10$  $\frac{dA}{dx} = 2\left(\frac{2}{2x}\right) - 1 = \frac{2}{x} - 1 = 0 \Rightarrow x = 2$ is the only critical value. For $x = 2,\ A = 2\ln(2 \cdot 2) - 2 + 10 = 2\ln 4 + 8.$ Since there is only one critical value and it is a local maximum by the Second Derivative Test, it must be the absolute maximum.

(D)	This option is incorrect. This is the area of the triangle with the third vertex at the endpoint for $x = 8$: $A = \frac{1}{2}bh = \frac{1}{2}(4)\left(\ln(2x) - \frac{1}{2}x + 5\right) = 2\ln(2x) - x + 10.$ For $x = 8,\ A = 2\ln(2 \cdot 8) - 8 + 10 = 2\ln 16 + 2.$

Question 29

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.1C Calculate derivatives.	**2.1C6:** The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 5:** Building notational fluency

(A)	This option is incorrect. A misconception that the tangent line for the inverse function is perpendicular to the tangent line for the function could have led to using a slope that is the "negative reciprocal", i.e., believing that $\frac{d}{dx}f^{-1}(x) = -\frac{1}{f'(x)}$: $f(x) = x^3 + 4x + 2$ $f'(x) = 3x^2 + 4$ $f'(2) = 3 \cdot 2^2 + 4 = 16.$
(B)	This option is incorrect. The notation $g(x) = f^{-1}(x)$ may have been confused with the reciprocal of $f(x)$, with the attempt then to find $\frac{d}{dx}\frac{1}{f(x)}$ by using the quotient rule (or a power rule along with the chain rule): $\frac{d}{dx}\left(\frac{1}{x^3 + 4x + 2}\right) = \frac{\left(x^3 + 4x + 2\right)(0) - (1)\left(3x^2 + 4\right)}{\left(x^3 + 4x + 2\right)^2}.$ For $x = 2$, this gives: $\frac{-\left(3 \cdot 2^2 + 4\right)}{\left(2^3 + 4 \cdot 2 + 2\right)^2} = -\frac{16}{18^2} = -\frac{4}{81}.$

(C)	**This option is correct.** Since $f(g(x)) = x,$ the chain rule can be used to determine that $f'(g(x))g'(x) = 1.$ Substituting $x = 2$ gives $1 = f'(g(2))g'(2) = f'(0)g'(2).$ Therefore $g'(2) = \frac{1}{f'(0)} = \frac{1}{3 \cdot 0^2 + 4} = \frac{1}{4}.$ Alternatively, one can use the following formula that follows from the same chain rule calculation as above when starting with $f\left(f^{-1}(x)\right) = x$: $\frac{d}{dx} f^{-1}(x)\Big\|_{x=2} = \frac{1}{f'\left(f^{-1}(2)\right)} = \frac{1}{f'(0)} = \frac{1}{3 \cdot 0^2 + 4} = \frac{1}{4}.$
(D)	This option is incorrect. An attempt might have been made to use the memorized formula for the derivative of an inverse function shown in the solution in option (C), but taking the reciprocal was forgotten, leading to computing $f'\left(f^{-1}(2)\right)$: $f'(x) = 3x^2 + 4$ $f'\left(f^{-1}(2)\right) = f'(0) = 3 \cdot 0^2 + 4 = 4.$

Question 30

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.2A(a) Interpret the definite integral as the limit of a Riemann sum.	**3.2A3:** The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 5:** Building notational fluency

(A)	This option is incorrect. This sum used rectangles of width $\Delta x = \frac{1}{n}$ and right endpoints corresponding to step sizes of width $\frac{1}{n}$. These would be the values for a right Riemann sum on the interval $[2, 3]$. This limit is therefore $\int_2^3 x^2\, dx$.
(B)	This option is incorrect. This sum used step sizes of width $\frac{1}{n}$ between the right endpoints which is not consistent with the width $\frac{3}{n}$ used for the rectangles. This limit is $\int_2^3 3x^2\, dx$.
(C)	This option is incorrect. This sum used step sizes of width $\frac{3}{n}$ between the right endpoints which is not consistent with the width $\frac{1}{n}$ used for the rectangles. This limit is $\int_2^3 \frac{1}{3}x^2\, dx$.
(D)	**This option is correct.** For this integral, a right Riemann sum with n terms is built from rectangles of width $\Delta x = \frac{5-2}{n} = \frac{3}{n}$. The height of each rectangle is determined by the function x^2 evaluated at the right endpoints of the n intervals. These endpoints are the x-values at $2 + k\Delta x$, for k from 1 to n, since the integral starts at 2. The Riemann sum can be written as $\sum_{k=1}^{n}(2 + k\Delta x)^2\,\Delta x$ where $\Delta x = \frac{3}{n}$, and its limit as $n \to \infty$ is $\int_2^5 x^2\, dx$.