Question 76

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.4E Use the definite integral to solve problems in various contexts.	**3.4E1:** The definite integral can be used to express information about accumulation and net change in many applied contexts.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/ computational processes

(A)	**This option is correct.** The definite integral of the rate of change over an interval gives the net change over that interval. Since $s(t)$ is the rate of change at which sand is added to the beach, the definite integral of $s(t)$ on the interval $2 \le t \le 5$ gives the net change in the amount of sand on the beach over that time period. Since t is measured in hours since 5:00 A.M., the time 7:00 A.M. corresponds to $t = 2$ and 10:00 A.M. corresponds to $t = 5$. $\int_2^5 s(t)\,dt = \int_2^5 (65 + 24\sin(0.3t))\,dt = 255.368$
(B)	This option is incorrect. The definite integral was correctly used to find a net change given a rate of change, but incorrectly uses the 3-hour period from $t = 0$ to $t = 3$ by not adjusting for the variable t representing the time since 5:00 A.M. $\int_0^3 s(t)\,dt$
(C)	This option is incorrect. This is the average value of the rate at which sand is being added over the interval $2 \le t \le 5$: $\dfrac{\int_2^5 s(t)\,dt}{5 - 2}.$
(D)	This option is incorrect. This is the net change $s(5) - s(2)$ in the rate at which sand is being added over the interval $2 \le t \le 5$, not the net change in the amount of sand on the beach over that interval.

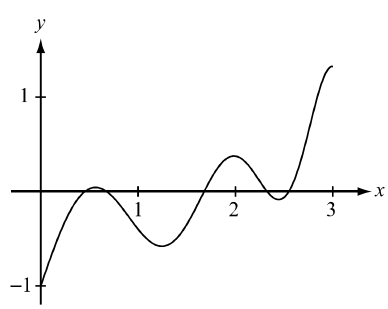
Question 77

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
1.1B Estimate limits of functions.	**1.1B1:** Numerical and graphical information can be used to estimate limits.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 4:** Connecting multiple representations

(A)	This option is incorrect. The value $a = 4$ is missed, perhaps because of a mistaken belief that the function must be continuous at the point for the limit to exist.
(B)	**This option is correct.** $\lim_{x \to a} f(x) = 0$ if the left-sided and right-sided limits at $x = a$ both equal 0, i.e. $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = 0.$ For this graph, that happens at $x = 2$ and $x = 4.$ Note that the function does not need to be defined at $x = 4$ for the limit to exist there.
(C)	This option is incorrect. These are the values for which $f(x) = 0$ and at least a one-sided limit exists. However, $\lim_{x \to a} f(x)$ does not exist for $a = 0.$
(D)	This option is incorrect. These are the values where $f(x) = 0.$ However, $\lim_{x \to a} f(x)$ does not exist for $a = 0$ and the limit is equal to $-1,$ not 0, for $a = 1.$

Question 78

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.2A Use derivatives to analyze properties of a function.	**2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 2:** Connecting concepts

(A)	This option is incorrect. This might be selected if a badly chosen viewing window (i.e. not examining the correct interval) omits the first two and last two intersections.
(B)	This option is incorrect. This might be selected if the viewing window omits the first two intersections or the last two intersections, or if the first two intersections are overlooked, perhaps because the range on the y-axis is set too large.
(C)	This option is incorrect. This is the apparent number of points of inflection on the graph of f''. It is also the number of critical points of f'' on the open interval $0 < x < 3$.
(D)	**This option is correct.** Graph $f''(x)$ on the interval $0 \le x \le 3$.  A point of inflection occurs where the second derivative $f''(x)$ changes sign. This happens five times on the interval: twice between 0 and 1, once between 1 and 2, and twice between 2 and 3.

Question 79

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.4C Apply definite integrals to problems involving motion.	**3.4C1:** For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.	**MPAC 4:** Connecting multiple representations **MPAC 1:** Reasoning with definitions and theorems

(A)	This option is incorrect. This is the difference between the displacement and the total distance over the whole interval $0 \le t \le 5$.
(B)	**This option is correct.** Let the total area above the t-axis be A and the area below the t-axis be B. The particle's total distance on the time interval $0 \le t \le 5$ is: $\int_0^5 \lvert v(t) \rvert \, dt = A + B = 13,$ and the particle's displacement is: $\int_0^5 v(t) \, dt = A - B = 3.$ Solving the system gives $A = 8$ and $B = 5$. Since the graph of $v(t)$ is below the axis for $2 < t < 4$, $\int_2^4 v(t) \, dt = -B = -5.$
(C)	This option is incorrect. $\int_2^4 v(t) \, dt = B = 5$ was concluded by thinking just about area and not that the definite integral corresponds to a region below the axis.
(D)	This option is incorrect. This is the difference between the total distance and the displacement over the whole interval $0 \le t \le 5$.

Question 80

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.4A Interpret the meaning of a definite integral within a problem.	**3.4A2:** The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.	**MPAC 2:** Connecting concepts **MPAC 5:** Building notational fluency

(A)	This option is incorrect. If $T(t)$ was the room temperature, this would be the interpretation for the expression $T(0)+\int_0^6 H(t)\,dt$ that would be used to find the temperature at $t=6$ given the initial value of 20.
(B)	This option is incorrect. If $T(t)$ was the room temperature, this would be an interpretation for the expression $\frac{1}{6}\int_0^6 T(t)\,dt$.
(C)	**This option is correct.** The definite integral of the rate of change over an interval gives the net change over that interval. Since $H(t)$ is the rate of change of temperature, the definite integral of $H(t)$ on the interval $0 \le t \le 6$ gives the change in the temperature over that time period.
(D)	This option is incorrect. This is an interpretation of $H(6)$.

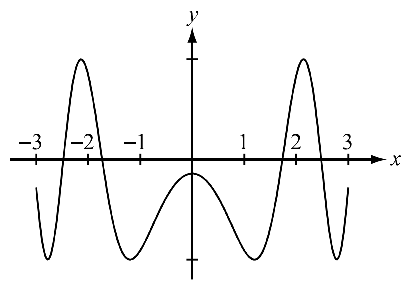
Question 81

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.2A Use derivatives to analyze properties of a function.	**2.2A3:** Key features of the graphs of f, f', and f'' are related to one another.	**MPAC 4:** Connecting multiple representations **MPAC 2:** Connecting concepts

(A)	**This option is correct.** Where the derivative f' is positive, the function f should be increasing. Where the derivative f' is negative, the function f should be decreasing. The derivative to the left of $x = 0$ is always positive. The derivative to the right of $x = 0$ is positive until $x = 1,$ then negative. Therefore the graph of f should be increasing for $x < 0$ and $0 < x < 1$, and should be decreasing for $x > 1.$ The graph in this option could therefore be the graph of f.
(B)	This option is incorrect. The sign of the derivative on the left side and right side of $x = 0$ was misinterpreted, thus reversing the desired increasing/decreasing behavior that the graph of f should display.
(C)	This option is incorrect. The sign of the derivative on the left side of $x = 0$ was misinterpreted, thus reversing the desired increasing behavior that the graph of f should display on the left side of $x = 0$.
(D)	This option is incorrect. The sign of the derivative on the right side of $x = 0$ was misinterpreted, thus not achieving the desired increasing/decreasing behavior that the graph of f should display on the right side of $x = 0$.

Question 82

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.2A Use derivatives to analyze properties of a function.	**2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 2:** Connecting concepts

(A)	**This option is correct.** The graph of f' on the interval $-3 < x < 3$ shows that the function f will have a relative maximum where the derivative changes from positive to negative. This happens between -2 and -1, and between 2 and 3. The calculator is used to find the zeros of f' on those intervals. 
(B)	This option is incorrect. These are the values where the derivative changes from negative to positive. They would be where f has a relative minimum, not a relative maximum.
(C)	This option is incorrect. These are the values where the derivative has a relative maximum, not where the function has a relative maximum.
(D)	This option is incorrect. These are all of the values where the derivative is equal to zero. They are critical values, but the change in the sign of the derivative has not been used to distinguish between a relative maximum and a relative minimum.

Question 83

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
1.2A Analyze functions for intervals of continuity or points of discontinuity.	**1.2A3:** Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.	**MPAC 4:** Connecting multiple representations **MPAC 1:** Reasoning with definitions and theorems

(A)	This option is incorrect. This might be chosen because the function f is not differentiable at $x = 1.$ However, it is continuous at this point.
(B)	This option is incorrect. The type of discontinuity was misidentified. Since $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 0,$ there is no jump discontinuity at $x = 3.$ The function has a removable discontinuity at $x = 3.$
(C)	**This option is correct.** A jump discontinuity occurs at $x = c$ if the left- and right-sided limits exist at c, but are not equal, that is, $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x).$ For this function, $\lim_{x \to 7^-} f(x) = 4$ $\lim_{x \to 7^+} f(x) = 1$ and so there is a jump discontinuity at $x = 7.$
(D)	This option is incorrect. The type of discontinuity was misidentified. Since $\lim_{x \to 10^-} f(x) = \lim_{x \to 10^+} f(x) \approx 0.5,$ there is no jump discontinuity at $x = 10.$ The function is defined at $x = 10$ and has a removable discontinuity there.

Question 84

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
3.3B(b) Evaluate definite integrals.	**3.3B2:** If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes

(A)	This option is incorrect. A misunderstanding of the Fundamental Theorem of Calculus resulted in computing $f(1) + \int_1^5 f''(x)\,dx$.
(B)	This option is incorrect. Computing $f(1) + f'(5)$ shows a misunderstanding of the Fundamental Theorem of Calculus.
(C)	This option is incorrect. A common error is to neglect to use the initial condition and just calculate $f(5) = \int_1^5 \sqrt{x^3 + 6}\,dx$. This may arise from a misunderstanding of the Fundamental Theorem of Calculus or just forgetting to add the initial condition after computing the definite integral with the calculator.
(D)	**This option is correct.** By the Fundamental Theorem of Calculus, $f(5) - f(1) = \int_1^5 f'(x)\,dx$. Therefore $f(5) = f(1) + \int_1^5 \sqrt{x^3 + 6}\,dx$ $= \pi + 24.672 \approx 27.814$.

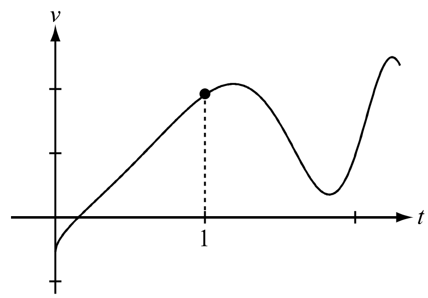
Question 85

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.3D Solve problems involving rates of change in applied contexts.	**2.3D1:** The derivative can be used to express information about rates of change in applied contexts.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 2:** Connecting concepts

(A)	This option is incorrect. The focus was only on the rate at which people enter the building. This inequality says that the total number of people who have entered the building is increasing.
(B)	This option is incorrect. Here the focus is only on the rate at which people enter the building. This inequality says that the rate of change of the number of people entering the building is increasing.
(C)	This option is incorrect. The expression $R(t) = f(t) - g(t)$ indicates the rate of change of the number of people in the building (rate in minus rate out). The sign of $R(t)$ determines whether the number of people in the building is increasing or decreasing at time t. This inequality says that the number of people in the building is increasing.
(D)	**This option is correct.** The expression $R(t) = f(t) - g(t)$ indicates the rate of change of the number of people in the building (rate in minus rate out). If the derivative $R'(t) = f'(t) - g'(t)$ is positive, then $R(t)$ is increasing, i.e. the rate of change of the number of people in the building is increasing.

Question 86

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.3C Solve problems involving related rates, optimization, rectilinear motion, (BC) and planar motion.	**2.3C1:** The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 2:** Connecting concepts

(A)	This option is incorrect. The meaning of the sign of the velocity may have been misinterpreted.
(B)	**This option is correct.** At $t = 1$, the graph of v is positive. In addition, the graph of v is increasing at $t = 1$, which means that $v'(1)$ is positive. Alternatively, $v(1) = 1.912$ and $v'(1) = 1.617$.  Since velocity is the derivative of position, a positive value of v at $t = 1$ means that x is increasing, i.e. the particle is moving to the right. Since acceleration is the derivative of velocity, a positive value of v' at $t = 1$ means that the particle is moving with positive acceleration.
(C)	This option is incorrect. Both claims are reversed.
(D)	This option is incorrect. The meaning of the sign of the derivative of velocity may have been misinterpreted.

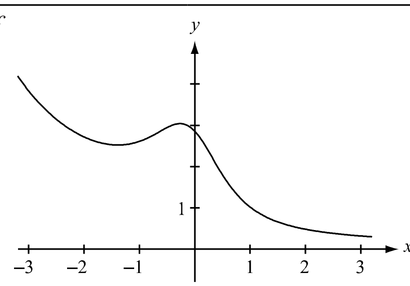
Question 87

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.3A Interpret the meaning of a derivative within a problem.	**2.3A1:** The unit for $f'(x)$ is the unit for f divided by the unit for x.	**MPAC 5:** Building notational fluency **MPAC 3:** Implementing algebraic/computational processes

(A)	This option is incorrect. This is the units of the dependent variable only.
(B)	This option is incorrect. This is the units of the independent variable only.
(C)	This option is incorrect. The dependent and independent variables were inverted. While in many problems time is often the independent variable, in this model time is specified as a function of the air pressure and hence is the dependent variable. That is why it is important to understand the functional notation $t = f(p)$.
(D)	**This option is correct.** The derivative of the function $t = f(p)$ is the limit of the difference quotient $\frac{\Delta t}{\Delta p}$. The units on the dependent variable, t, in the numerator is hours and the units on the independent variable, p, in the denominator is psi. The rate of change, therefore, has units in hours per psi.

Question 88

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
2.2A Use derivatives to analyze properties of a function.	**2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 2:** Connecting concepts

(A)	This option is incorrect. A common error is to answer the question as if the graph drawn on the calculator were the graph of the function f, not the graph of the derivative f'. The function f' is increasing between its local minimum and its local maximum, and this interval appears to match the location of those two points. The precise values can be found by solving for where $f''(x) = 0$.
(B)	This option is incorrect. There may have been confusion about which property of the derivative provides information about where the function is increasing. If one believes that the desired property is where the graph of f' is concave up, then these two intervals appear to work. The precise values of the endpoints can be found by solving for where $f'''(x) = 0$.
(C)	**This option is correct.** The function f will be increasing where f' is positive. Graph f' on an interval that includes at least $-2 \le x \le 1$ since that covers the choices in options (A) and (B).  The graph of f' is always positive on the interval in the window. The graph therefore rules out options (A), (B), and (D). The best choice is therefore (C). [One cannot verify graphically that f' is positive for $-\infty < x < \infty$. The denominator in the formula for f' is always positive. The numerator is clearly positive for $x \ge 0$. For $x < 0$, observe that the graph of $2e^{-x}$ is concave up and therefore lies above the tangent line at $x = 0$. This tangent line is $y = 2 - 2x$. Hence $2e^{-x} > 2 - 2x > -x$ since $x < 0 < 2$.]
(D)	This option is incorrect. There may have been confusion about which sign of the derivative provides information about the increasing behavior of the function. Or the observation that there are no zeros of f' might incorrectly suggest that there are no intervals to investigate.

Question 89

Learning Objective	Essential Knowledge	Mathematical Practices for AP Calculus
1.2B Determine the applicability of important calculus theorems using continuity.	**1.2B1:** Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 4:** Connecting multiple representations

(A)	This option is incorrect. The number of times that $f(x) = 4$ may have been confused with the number of times that $x = 4$, or it may have been thought that the Intermediate Value Theorem only applies to the interval $[a, b]$ when $f(a) \ge 4 \ge f(b)$, namely on $[4, 6]$.
(B)	This option is incorrect. It may have been thought that the Intermediate Value Theorem only applies to the interval $[a, b]$ when $f(a) \le 4 \le f(b)$, namely on $[0, 4]$ and $[8, 13]$.
(C)	**This option is correct.** Because f is continuous, the Intermediate Value Theorem will imply that $f(x)$ will equal 4 on an interval $[a, b]$ where 4 is between $f(a)$ and $f(b)$. According to the table, this happens for the intervals $[0, 4]$, $[4, 6]$, and $[8, 13]$. So there are at least three values of x for which the Intermediate Value Theorem applies and $f(x) = 4$.
(D)	This option is incorrect. It may have been thought that an additional value of x occurs on $[6, 13]$ because $f(6) < 4 < f(13)$. But this x-value may be the same as the one guaranteed by the Intermediate Value Theorem on the interval $[8, 13]$.

Question 90

Learning Objective		Essential Knowledge	Mathematical Practices for AP Calculus
2.2B Recognize the connection between differentiability and continuity.		**2.2B2:** If a function is differentiable at a point, then it is continuous at that point.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 4:** Connecting multiple representations
(A)	This option is incorrect. It does not recognize that statement II is true.		
(B)	**This option is correct.** By the Fundamental Theorem of Calculus, $h(3) = h(-1) + \int_{-1}^{3} h'(x)\,dx.$ Since the region between the graph of h' and the x-axis has more area above the axis than below the axis, the definite integral $\int_{-1}^{3} h'(x)\,dx$ is positive. Therefore $h(3) > h(-1)$ and statement I is false. The graph shows that derivative of h exists on the interval $(-1, 3).$ That means that the function h is differentiable on $(-1, 3)$ and is therefore continuous on that interval. In particular, h is continuous at $x = 1,$ so statement II must be true. Because h is continuous at $x = 1$ and $h'(1) \approx 0.5,$ the graph of h has a tangent line at $x = 1$ that is not vertical. Locally near $x = 1$ the graph of h will look like that tangent line and thus there cannot be a vertical asymptote at $x = 1.$ Therefore statement III is false.		
(C)	This option is incorrect. Statement I might be chosen by interpreting the given graph as the graph of h rather than its derivative. It might also be chosen by recognizing that $h(3) - h(-1) = \int_{-1}^{3} h'(x)\,dx$ but believing that the symmetry of the areas within the semicircles results in a net change of 0 for h over the interval $-1 \le x \le 3.$		
(D)	This option is incorrect. Statement I might be chosen for the same reasons as in option (C). Statement III might be chosen by interpreting the given graph as the graph of h rather than its derivative and confusing vertical asymptote with vertical tangent, or by thinking that the vertical tangent on h' at $x = 1$ results in a vertical asymptote on h at $x = 1.$		