A: Integrate using a u-substitution.

#1) $\int (x^2-1)^5 2x\,dx$ $= \int u^5 \cancel{2x}\left(\frac{du}{\cancel{2x}}\right)$

$u = x^2 - 1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$= \int u^5\,du$

$= \frac{1}{6}u^6 + C$

$= \frac{1}{6}(x^2-1)^6 + C$

#2) $\int e^{x^4} 4x^3\,dx$ $= \int e^u \cancel{4x^3}\left(\frac{du}{\cancel{4x^3}}\right)$

$u = x^4$

$\frac{du}{dx} = 4x^3$

$du = 4x^3\,dx$

$\frac{du}{4x^3} = dx$

$= \int e^u\,du$

$= e^u + C$

$= e^{x^4} + C$

#3) $\int \frac{5x^4}{x^5-9}\,dx$ $= \int \frac{\cancel{5x^4}}{u}\left(\frac{du}{\cancel{5x^4}}\right)$

$u = x^5 - 9$

$\frac{du}{dx} = 5x^4$

$du = 5x^4\,dx$

$\frac{du}{5x^4} = dx$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln|x^5-9| + C$

#4) $\int (x^2-1)^5 x\,dx$ $= \int u^5 \cancel{x}\left(\frac{du}{2\cancel{x}}\right)$

$u = x^2 - 1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$= \frac{1}{2}\int u^5\,du$

$= \frac{1}{2}\cdot\frac{1}{6}u^6 + C$

$= \frac{1}{12}(x^2-1)^6 + C$

#5) $\int e^{x^4} 7x^3\,dx$ $= \int e^u 7\cancel{x^3}\left(\frac{du}{4\cancel{x^3}}\right)$

$= \frac{7}{4}\int e^u\,du$

$= \frac{7}{4}e^u + C$

$= \frac{7}{4}e^{x^4} + C$

$u = x^4$
$\frac{du}{dx} = 4x^3$
$du = 4x^3dx$
$\frac{du}{4x^3} = dx$

#6) $\int \frac{x^4}{x^5-9}\,dx$ $= \int \frac{\cancel{x^4}}{u}\left(\frac{du}{5\cancel{x^4}}\right)$

$= \frac{1}{5}\int \frac{1}{u}\,du$

$= \frac{1}{5}\ln|u| + C$

$= \frac{1}{5}\ln|x^5-9| + C$

$u = x^5 - 9$
$\frac{du}{dx} = 5x^4$
$du = 5x^4dx$
$\frac{du}{5x^4} = dx$

B: If possible, integrate by a u-substitution. If not possible, say so.

#7) $\int (x^4-9)^3x^3\,dx$ $= \int u^3 \cancel{x^3}\left(\frac{du}{4\cancel{x^3}}\right)$

$= \frac{1}{4}\int u^3du$

$= \frac{1}{4}\cdot\frac{1}{4}u^4 + C$

$= \frac{1}{16}(x^4-9)^4 + C$

$u = x^4 - 9$
$\frac{du}{dx} = 4x^3$
$du = 4x^3dx$
$\frac{du}{4x^3} = dx$

#8) $\int (x^4-9)^3x^5\,dx$ $= \int u^3 \overset{x^2}{\cancel{x^5}}\left(\frac{du}{4\cancel{x^3}}\right)$

$u = x^4 - 9$
$\frac{du}{dx} = 4x^3$
$du = 4x^3dx$
$\frac{du}{4x^3} = dx$

CAN'T INTEGRATE BY SUBSTITUTION

#9) $\int e^{x^3} x^3 \, dx$ $= \int e^u x^{\cancel{3}} \left(\frac{du}{3\cancel{x^2}}\right)$

$= \int e^u x \, du$

$\boxed{\begin{array}{l} u = x^3 \\ \frac{du}{dx} = 3x^2 \\ du = 3x^2 dx \\ \frac{du}{3x^2} = dx \end{array}}$

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#10) $\int e^{z^2} 3z \, dz$ $= \int e^u 3\cancel{z} \left(\frac{du}{2\cancel{z}}\right)$

$\boxed{\begin{array}{l} u = z^2 \\ \frac{du}{dz} = 2z \\ du = 2z \, dz \\ \frac{du}{2z} = dz \end{array}}$

$= \frac{3}{2} \int e^u \, du$

$= \frac{3}{2} e^u + C$

$= \frac{3}{2} e^{z^2} + C$

#11) $\int \frac{z^2}{5z^3+1} \, dz$ $= \int \frac{\cancel{z^2}}{u} \left(\frac{du}{15\cancel{z^2}}\right)$

$\boxed{\begin{array}{l} u = 5z^3 + 1 \\ \frac{du}{dz} = 15z^2 \\ du = 15z^2 dz \\ \frac{du}{15z^2} = dz \end{array}}$

$= \frac{1}{15} \int \frac{1}{u} \, du$

$= \frac{1}{15} \ln|u| + C$

$= \frac{1}{15} \ln|5z^3+1| + C$

#12) $\int \frac{dz}{2z+1}$ $= \int \frac{\left(\frac{du}{2}\right)}{u}$

$\boxed{\begin{array}{l} u = 2z + 1 \\ \frac{du}{dz} = 2 \\ du = 2dz \\ \frac{du}{2} = dz \end{array}}$

$= \frac{1}{2} \int \frac{1}{u} \, du$

$= \frac{1}{2} \ln|u| + C$

$= \frac{1}{2} \ln|2z+1| + C$

#13) $\int \sqrt[5]{t^4+81}\, t^3\, dt$ $= \int u^{\frac{1}{5}} \cancel{t^3} \left(\frac{du}{4\cancel{t^3}}\right)$

$u = t^4+81$
$\frac{du}{dt} = 4t^3$
$du = 4t^3 dt$
$\frac{du}{4t^3} = dt$

$= \frac{1}{4}\int u^{\frac{1}{5}} du$

$= \frac{1}{4}\left(\frac{5}{6}\right)u^{6/5} + C$

$= \frac{5}{24}\left(\sqrt[5]{t^4+81}\right)^6 + C$

#14) $\int \sqrt[3]{t^5-1}\, t^3\, dt$ CAN'T INTEGRATE BY SUBSTITUTION

$u = t^5-1$
$\frac{du}{dt} = 5t^4$
$du = 5t^4 dt$

#15) $\int (3t^2+6t)^3(6t+6)\, dt$

$u = 3t^2+6t$
$\frac{du}{dt} = 6t+6$
$du = (6t+6)dt$
$\frac{du}{6t+6} = dt$

$= \int u^3 \cancel{(6t+6)} \left(\frac{du}{\cancel{6t+6}}\right)$

$= \int u^3 du$

$= \frac{1}{4}u^4 + C$

$= \frac{1}{4}(3t^2+6t)^4 + C$

#16) $\int (x^2+5x)^3(4x+10)\, dx$

$u = x^2+5x$
$\frac{du}{dx} = 2x+5$
$du = (2x+5)dx$
$\frac{du}{2x+5} = dx$

$= \int u^3\, 2\cancel{(2x+5)} \left(\frac{du}{\cancel{2x+5}}\right)$

$= 2\int u^3 du$

$= 2\left(\frac{1}{4}\right)u^4 + C$

$= \frac{1}{2}(x^2+5x)^4 + C$

#17) $\int \frac{4x^3+3x^2}{x^4+x^3}\,dx$

$$= \int \frac{\cancel{4x^3+3x^2}}{u}\left(\frac{du}{\cancel{4x^3+3x^2}}\right)$$

$u = x^4 + x^3$

$\frac{du}{dx} = 4x^3 + 3x^2$

$du = (4x^3+3x^2)dx$

$\frac{du}{4x^3+3x^2} = dx$

$$= \int \frac{1}{u}\,du$$

$$= \ln|u| + C$$

$$= \ln|x^4+x^3| + C$$

#18) $\int \frac{20x^4+6x}{2x^5+x^2}\,dx$

$$= \int \frac{2(10x^4+3x)}{u}\left(\frac{du}{10x^4+2x}\right)$$

$u = 2x^5 + x^2$

$\frac{du}{dx} = 10x^4 + 2x$

$du = (10x^4+2x)dx$

$\frac{du}{10x^4+2x} = dx$

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#19) $\int e^{x^3+x}(6x^2+2)\,dx$

$$= \int e^u\, 2(\cancel{3x^2+1})\left(\frac{du}{\cancel{3x^2+1}}\right)$$

$u = x^3 + x$

$\frac{du}{dx} = 3x^2 + 1$

$du = (3x^2+1)dx$

$\frac{du}{3x^2+1} = dx$

$$= 2\int e^u\,du$$

$$= 2e^u + C$$

$$= 2e^{x^3+x} + C$$

#20) $\int (2x-4)^5\,dx$

$$= \int u^5\left(\frac{du}{2}\right)$$

$u = 2x - 4$

$\frac{du}{dx} = 2$

$du = 2\,dx$

$\frac{du}{2} = dx$

$$= \frac{1}{2}\int u^5\,du$$

$$= \frac{1}{2}\left(\frac{1}{6}u^6\right) + C$$

$$= \frac{1}{12}(2x-4)^6 + C$$

#21) $\int \frac{e^x}{e^x-1}\,dx$ $= \int \frac{\cancel{e^x}}{u}\left(\frac{du}{\cancel{e^x}}\right)$

$$\boxed{\begin{aligned} u &= e^x - 1 \\ \frac{du}{dx} &= e^x \\ du &= e^x dx \\ \frac{du}{e^x} &= dx \end{aligned}}$$

$= \int \frac{du}{u}$

$= \ln|u| + C$

$= \ln|e^x - 1| + C$

#22) $\int \frac{\ln x}{x}\,dx$ (Pro tip: set $u = \ln x$)

$$\boxed{\begin{aligned} u &= \ln x \\ \frac{du}{dx} &= \frac{1}{x} \\ du &= \frac{1}{x}dx \\ x\,du &= dx \end{aligned}}$$

$= \int \frac{u}{\cancel{x}}(\cancel{x}\,du)$

$= \int u\,du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}(\ln x)^2 + C$

$= \frac{1}{2}\ln^2 x + C$

#23) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ (Pro tip: set $u = \sqrt{x}$)

$= \int \frac{e^u}{\cancel{\sqrt{x}}}(2\cancel{\sqrt{x}}\,du)$

$= 2\int e^u\,du$

$= 2e^u + C$

$= 2e^{\sqrt{x}} + C$

$$\boxed{\begin{aligned} u &= \sqrt{x} \\ \frac{du}{dx} &= \frac{1}{2}x^{-\frac{1}{2}} \\ du &= \frac{1}{2\sqrt{x}}dx \\ 2\sqrt{x}\,du &= dx \end{aligned}}$$

C: Find each integral by using algebra first.

#24) $\int (x+9)x^3\,dx$ $= \int (x^4 + 9x^3)\,dx$

$= \frac{1}{5}x^5 + \frac{9}{4}x^4 + C$

#25) $\int (x+3)^2 5x\,dx = \int (x^2+6x+9)\,5x\,dx$

$= \int (5x^3 + 30x^2 + 45x)\,dx$

$= \frac{5}{4}x^4 + 10x^3 + \frac{45}{2}x^2 + C$

#26) $\int (x-1)^2\sqrt{x}\,dx$

$\int (x-1)^2\sqrt{x}\,dx = \int (x^2 - 2x + 1)\,x^{\frac{1}{2}}\,dx$

$= \int (x^{\frac{4}{2}} - 2x^{\frac{2}{2}} + 1)\,x^{\frac{1}{2}}\,dx$

$= \int (x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + x^{\frac{1}{2}})\,dx$

$= \frac{2}{7}x^{\frac{7}{2}} - 2(\frac{2}{5})x^{5/2} + \frac{2}{3}x^{\frac{3}{2}} + C$

$= x^{\frac{1}{2}}\left(\frac{2}{7}x^{\frac{6}{2}} - \frac{4}{5}x^{\frac{4}{2}} + \frac{2}{3}x^{\frac{2}{2}}\right) + C$

$= \sqrt{x}\left(\frac{2}{7}x^3 - \frac{4}{5}x^2 + \frac{2}{3}x\right) + C$

#27) $\int (x+4)(x-4)\,dx = \int (x^2 - 16)\,dx$

$= \frac{1}{3}x^3 - 16x + C$