Basic Integration

10.3 – U Substitution Definite Integrals

Evaluating Definite Integrals by Substitution

#1) $\int_4^5 \frac{dx}{3-x} = \int_{-1}^{-2} \frac{-du}{u}$

$u = 3-x$

$\frac{du}{dx} = -1$

$du = -dx$

$-du = dx$

$= \int_{-2}^{-1} \frac{1}{u}\, du$

$= \ln|u| \Big|_{-2}^{-1}$

$= [\ln|-1|] - [\ln|-2|]$

$= \ln 1 - \ln 2$

$= 0 - \ln 2$

$= -\ln 2$

Marginal Butter

#1) *I Can't Believe It's Not Butter Inc's* marginal (wink, wink) cost function is $MC(x) = \frac{6x^2}{x^3+1}$ and fixed costs are \$1000. Find the cost function.

$C(x) = \int \frac{6x^2}{x^3+1}\, dx$

$= \int \frac{6x^2}{u}\left(\frac{du}{3x^2}\right)$

$= 2\int \frac{1}{u}\, du$

$= 2\ln|u| + C$

$C(x) = 2\ln|x^3+1| + C$

$1000 = 2\ln|(0)^3+1| + C$

$1000 = 2\ln|1| + C$

$1000 = 2(0) + C$

$1000 = C$

$C(x) = 2\ln|x^3+1| + 1000$

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\, dx$

$\frac{du}{3x^2} = dx$

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George's Chuck

#2) Frogs are being chucked into a lake by George at the rate of $r(t) = 200te^{t^2}$ per year, where t is the number of years since the Great Frog Shortage of '15. Find the total number of frogs chucked into the lake during the first 2 years.

$$\text{Total Frogs} = \int_0^2 2000t\,e^{t^2}\,dt$$

$u = t^2$

$\frac{du}{dt} = 2t$

$du = 2t\,dt$

$\frac{1}{2t}\,du = dt$

$$= \int_0^4 2000t\,e^{u}\,\frac{1}{2t}\,du$$

$$= 1000\int_0^4 e^u\,du$$

$$= 1000e^u\Big|_0^4$$

$$= [1000e^4] - [1000e^0]$$

$$= (1000e^4 - 1000)\text{ Frogs}$$

$$\approx 53{,}598$$

George chucked about 53,598 frogs into the lake during the first two years

No Rainbows

#3) Because of the sheer volume of frogs in the lake, it began to overflow. After x minutes of the lake overflowing, the water level in George's basement is $L(x) = 40x(x^2+9)^{-1/2}$ inches. Find the average depth during the first 4 minutes.

$u = x^2 + 9$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$\text{Average Depth} = \frac{1}{4-0}\int_9^{25} \frac{40x}{\sqrt{x^2+9}}\,dx$$

$$= \frac{1}{4}\int_9^{25} \frac{40x}{\sqrt{u}}\left(\frac{du}{2x}\right)$$

$$= 5\int_9^{25} u^{-\frac{1}{2}}\,du$$

$$= 10\,u^{\frac{1}{2}}\Big|_9^{25}$$

$$= [10\sqrt{25}] - [10\sqrt{9}]$$

$$= [10(5)] - [10(3)]$$

$$= 50 - 30$$

$$\text{Average Depth} = 20$$

The average depth during the first 4 minutes is 20 inches.