

# Basic Integration

## 10.4 – Integration Substitution & Trig

A: Integrate the trig functions by substitution when needed

#1)  $\int 3 \cos(4x) dx$

$$= \int 3 \cos(u) \left(\frac{du}{4}\right)$$

$$= \frac{3}{4} \int \cos(u)$$

$$= \frac{3}{4} \sin(u) + C$$

$$= \frac{3}{4} \sin(4x) + C$$

$$\begin{aligned} u &= 4x \\ \frac{du}{dx} &= 4 \\ du &= 4 dx \\ \frac{du}{4} &= dx \end{aligned}$$

#3)  $\int 4 \sec(4x) \tan(4x) dx$

$$= \int 4 \sec(u) \tan(u) \left(\frac{du}{4}\right)$$

$$= \int \sec(u) \tan(u) du$$

$$= \sec(u) + C$$

$$= \sec(4x) + C$$

$$\begin{aligned} u &= 4x \\ \frac{du}{dx} &= 4 \\ du &= 4 dx \\ \frac{du}{4} &= dx \end{aligned}$$

#2)  $\int \sin(4x) dx$

$$= \int \sin(u) \frac{du}{4}$$

$$= \frac{1}{4} \int \sin(u) du$$

$$= -\frac{1}{4} \cos(u) + C$$

$$= -\frac{1}{4} \cos(4x) + C$$

$$\begin{aligned} u &= 4x \\ \frac{du}{dx} &= 4 \\ du &= 4 dx \\ \frac{du}{4} &= dx \end{aligned}$$

#4)  $\int \cos(8x) dx$

$$= \int \cos(u) \left(\frac{du}{8}\right)$$

$$= \frac{1}{8} \int \cos(u) du$$

$$= \frac{1}{8} \sin(u) + C$$

$$= \frac{1}{8} \sin(8x) + C$$

$$\begin{aligned} u &= 8x \\ \frac{du}{dx} &= 8 \\ du &= 8 dx \\ \frac{du}{8} &= dx \end{aligned}$$

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#5)  $\int \cos(11x) \sqrt{\sin(11x)} dx$

$$= \int \cos(11x) u^{\frac{1}{2}} \frac{du}{11 \cos(11x)}$$

$$= \frac{1}{11} \int u^{\frac{1}{2}} du$$

$$= \frac{1}{11} \frac{2}{3} u^{\frac{3}{2}} + C$$

$$= \frac{2}{33} \sqrt{\sin^3(11x)} + C$$

$u = \sin(11x)$   
 $du = 11 \cdot \cos(11x) dx$   
 $\frac{du}{11 \cos(11x)} = dx$

#6)  $\int \sin\left(\frac{\pi}{3}x\right) dx$

$$= \int \sin(u) \frac{3}{\pi} du$$

$$= \frac{3}{\pi} \int \sin(u) du$$

$$= \frac{3}{\pi} (-\cos(u)) + C$$

$$= -\frac{3}{\pi} \cos\left(\frac{\pi}{3}x\right) + C$$

$u = \frac{\pi}{3}x$   
 $du = \frac{\pi}{3} dx$   
 $\frac{3}{\pi} du = dx$

#7)  $\int \csc(x) \cot(x) \csc^2(x) dx$

$$= \int \csc(x) \cot(x) u \cdot \frac{du}{-\cot(x)}$$

$$= -\int \csc(x) \cdot u du$$

$$= -\int \sqrt{u} u du$$

$$= -\int u^{\frac{3}{2}} du$$

$$= -\frac{2}{5} u^{\frac{5}{2}} + C$$

$$= -\frac{2}{5} \left(\sqrt{\csc^2(x)}\right)^5 + C$$

$$= -\frac{2}{5} \csc^5(x) + C$$

$u = \csc^2(x)$   
 $du = -2 \cot(x) \csc(x) dx$   
 $\frac{du}{-2 \cot(x)} = dx$

→  $u = \csc^2(x)$   
 $\sqrt{u} = \csc(x)$

#8)  $\int \frac{\sec(x)}{\cos(x)} dx$

$$= \int \sec^2(x) dx$$

$$= \tan(x) + C$$

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#9)  $\int 2x \cos(x^2) dx$

$$\begin{aligned}
 &= \int \cancel{2x} \cos(u) \left( \frac{du}{\cancel{2x}} \right) \\
 &= \int \cos(u) du \\
 &= \sin(u) + C \\
 &= \sin(x^2) + C
 \end{aligned}$$

$$\begin{aligned}
 u &= x^2 \\
 \frac{du}{dx} &= 2x \\
 du &= 2x dx \\
 \frac{du}{2x} &= dx
 \end{aligned}$$

#11)  $\int 3x^2 \sec(x^3) \tan(x^3) dx$

$$\begin{aligned}
 &= \int \cancel{3x^2} \sec(u) \tan(u) \left( \frac{du}{\cancel{3x^2}} \right) \\
 &= \int \sec(u) \tan(u) du \\
 &= \sec(u) + C \\
 &= \sec(x^3) + C
 \end{aligned}$$

$$\begin{aligned}
 u &= x^3 \\
 \frac{du}{dx} &= 3x^2 \\
 du &= 3x^2 dx \\
 \frac{du}{3x^2} &= dx
 \end{aligned}$$

#10)  $\int \sin(4x) dx$

$$\begin{aligned}
 &= \int \sin(u) \left( \frac{du}{4} \right) \\
 &= \frac{1}{4} \int \sin(u) du \\
 &= -\frac{1}{4} \cos(u) + C \\
 &= -\frac{1}{4} \cos(4x) + C
 \end{aligned}$$

$$\begin{aligned}
 u &= 4x \\
 \frac{du}{dx} &= 4 \\
 du &= 4 dx \\
 \frac{du}{4} &= dx
 \end{aligned}$$

#12)  $\int \frac{1}{27} \sin(27x) dx$

$$\begin{aligned}
 &= \int \frac{1}{27} \sin(u) \left( \frac{du}{27} \right) \\
 &= \frac{1}{729} \int \sin(u) du \\
 &= -\frac{1}{729} \cos(u) + C \\
 &= -\frac{1}{729} \cos(27x) + C
 \end{aligned}$$

$$\begin{aligned}
 u &= 27x \\
 \frac{du}{dx} &= 27 \\
 du &= 27 dx \\
 \frac{du}{27} &= dx
 \end{aligned}$$

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#13)  $\int \sec^2(x) \sqrt{\tan(x)} dx$

$$= \int \cancel{\sec^2(x)} \sqrt{u} \left( \frac{du}{\cancel{\sec^2(x)}} \right)$$

$$= \int u^{\frac{1}{2}} du$$

$$= \frac{2}{3} u^{\frac{3}{2}} + C$$

$$= \frac{2}{3} \sqrt{\tan^3(x)} + C$$

$$\begin{aligned} u &= \tan(x) \\ \frac{du}{dx} &= \sec^2(x) \\ du &= \sec^2(x) dx \\ \frac{du}{\sec^2(x)} &= dx \end{aligned}$$

#15)  $\int \tan(x) \ln |\sec(x)| dx$

$$= \int \cancel{\tan(x)} u \left( \frac{du}{\cancel{\tan(x)}} \right)$$

$$= \int u du$$

$$= \frac{1}{2} u^2 + C$$

$$= \frac{1}{2} \left[ \ln |\sec(x)| \right]^2 + C$$

$$\begin{aligned} u &= \ln |\sec(x)| \\ \frac{du}{dx} &= \frac{\sec'(x)}{\sec(x)} \\ \frac{du}{dx} &= \frac{\sec(x) \tan(x)}{\sec(x)} \\ \frac{du}{dx} &= \tan(x) \\ du &= \tan(x) dx \\ \frac{du}{\tan(x)} &= dx \end{aligned}$$

#14)  $\int \frac{1}{\pi} \csc^2(\pi x) dx$

$$= \int \frac{1}{\pi} \csc^2(u) \left( \frac{du}{\pi} \right)$$

$$= \frac{1}{\pi^2} \int \csc^2(u) du$$

$$= \frac{1}{\pi^2} (-\cot(u)) + C$$

$$= \frac{-1}{\pi^2} \cot(\pi x) + C$$

$$\begin{aligned} u &= \pi x \\ \frac{du}{dx} &= \pi \\ du &= \pi dx \\ \frac{du}{\pi} &= dx \end{aligned}$$

#16)  $\int \sec(x) \ln |\sec(x) + \tan(x)| dx$

$$= \int \cancel{\sec(x)} \cdot u \left( \frac{du}{\cancel{\sec(x)}} \right)$$

$$= \int u du$$

$$= \frac{1}{2} u^2 + C$$

$$= \frac{1}{2} \left[ \ln |\sec(x) + \tan(x)| \right]^2 + C$$

$$\begin{aligned} u &= \ln |\sec(x) + \tan(x)| \\ \frac{du}{dx} &= \sec(x) \\ du &= \sec(x) dx \\ \frac{du}{\sec(x)} &= dx \end{aligned}$$