Advanced Integration
10.5A – Differential Equations

Verify that the function y satisfies the given differential equation.

#1) $y = e^{2x} - 3e^x + 2$
$y'' - 3y' + 2y = 4$

$y' = 2e^{2x} - 3e^x$
$y'' = 4e^{2x} - 3e^x$

$y'' \quad -3y' \quad +2y \quad = 4$

$(4e^{2x} - 3e^x) - 3(2e^{2x} - 3e^x) + 2(e^{2x} - 3e^x + 2) = 4$

$4e^{2x} - 3e^x - 6e^{2x} + 9e^x + 2e^{2x} - 6e^x + 4 = 4$

$4 = 4$

#2) $y = ke^{ax} - \frac{b}{a}$ (a, b and k are constants)
$y' = ay + b$

$y' = ake^{ax}$

$ake^{ax} = a\left(ke^{ax} - \frac{b}{a}\right) + b$

$ake^{ax} = ake^{ax} - b + b$

$ake^{ax} = ake^{ax}$

Find the general solution of each differential equation or state that the differential equation is not separable.

#3) $y^2y' = 4x$

$y^2 \cdot \frac{dy}{dx} = 4x$

$y^2 dy = 4x\,dx$

$\int y^2 dy = \int 4x\,dx$

$\frac{1}{3}y^3 = 2x^2 + C$

$y^3 = 6x^2 + C$

$y = \sqrt[3]{6x^2 + C}$

#4) $y' = x + y$

$\frac{dy}{dx} = x + y$

$dy = (x + y)dx$

NOT Seperable

#5) $y' = 6x^2y$

$\frac{dy}{dx} = 6x^2y$

$dy = 6x^2y\,dx$

$\frac{1}{y}dy = 6x^2dx$

$\int \frac{1}{y}dy = \int 6x^2dx$

$\ln|y| = 2x^3 + C$

$\log_e y = 2x^3 + C$ (Log form)

$e^{2x^3+C} = y$ (Exp form)

$e^{2x^3} \cdot e^C = y$

$e^{2x^3} \cdot C = y$

$Ce^{2x^3} = y$

#6) $y' = \frac{y}{x}$

$\frac{dy}{dx} = \frac{y}{x}$

$dy = \frac{y}{x}dx$

$\frac{1}{y}dy = \frac{1}{x}dx$

$\int \frac{1}{y}dy = \int \frac{1}{x}dx$

$\ln|y| = \ln|x| + C$

$\log_e y = \ln|x| + C$ (Log form)

$e^{\ln|x|+C} = y$ (Exp form)

$e^{\ln x} \cdot e^C = y$

$x \cdot C = y$

$Cx = y$

#7) $yy' = 4x$

$y \cdot \frac{dy}{dx} = 4x$

$y\,dy = 4x\,dx$

$\int y\,dy = \int 4x\,dx$

$\frac{1}{2}y^2 = 2x^2 + C$

$y^2 = 4x^2 + C$

$y = \pm\sqrt{4x^2 + C}$

#8) $y' = e^{xy}$

$\frac{dy}{dx} = (e^x)^y$

$dy = (e^x)^y\,dx$

NOT Seperable

#9) $y' = 9x^2$

$\frac{dy}{dx} = 9x^2$

$dy = 9x^2 dx$

$\int dy = \int 9x^2 dx$

$y = 3x^3 + C$

#10) $y' = \frac{x}{x^2+1}$

$\frac{dy}{dx} = \frac{x}{x^2+1}$

$dy = \frac{x}{x^2+1} dx$

$\int dy = \int \frac{x}{x^2+1} dx$

$y = \int \frac{x}{u} \frac{1}{2x} du$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{1}{2x} du = dx$

$y = \frac{1}{2} \int \frac{1}{u} du$

$y = \frac{1}{2} \ln|u| + C$

$y = \frac{1}{2} \ln|x^2+1| + C$

#11) $y' = x^2 y$

$\frac{dy}{dx} = x^2 y$

$dy = x^2 y\, dx$

$\frac{1}{y} dy = x^2 dx$

$\int \frac{1}{y} dy = \int x^2 dx$

$\ln|y| = \frac{1}{3}x^3 + C$

$\log_e y = \frac{1}{3}x^3 + C$ (Log form)

$e^{\frac{1}{3}x^3 + C} = y$ (Exp Form)

$e^{\frac{1}{3}x^3} \cdot e^C = y$

$Ce^{\frac{1}{3}x^3} = y$

#12) $y' = x^m y^n \quad (m > 0, n \neq 1)$

$\frac{dy}{dx} = x^m y^n$

$dy = x^m y^n dx$

$y^{-n} dy = x^m dx$

$\int y^{-n} dy = \int x^m dx$

$\frac{1}{-n+1} y^{-n+1} = \frac{1}{m+1} x^{m+1} + C$

$y^{-n+1} = \frac{1-n}{m+1} x^{m+1} + C$

$y = \left(\frac{1-n}{m+1} x^{m+1} + C\right)^{\frac{1}{1-n}}$

#3) $y = \sqrt[3]{6x^2 + c}$

#4) Not separable

#5) $y = ce^{2x^3}$

#6) $y = cx$

#7) $y = \pm\sqrt{4x^2 + c}$

#8) Not separable

#9) $y = 3x^3 + c$

#10) $y = \frac{1}{2}\ln(x^2 + 1) + c$

#11) $y = ce^{\frac{x^3}{3}}$

#12) $y = \left(\frac{1-n}{m+1}x^{m+1} + c\right)^{\frac{1}{1-n}}$