George's Musk

#1) George's musk grows at rate *r* per year, and stench deposits are made at the rate of *d* body odor units per year. The value of his putridness *y(t)* after *t* years satisfies the differential equation $y' = d + ry$

Solve the differential equation with the deposit rate of *1000* body odor units and continuous growth rate of *0.05* with an initial condition of *y(0) = 0*.

$$\frac{dy}{dt} = 1000 + .05y$$

$$\frac{dy}{dt} = 1000 + \frac{1}{20}y$$

t = years
y = total pudridness
y' = net change in pudridness/year

$$dy = \left(1000 + \frac{1}{20}y\right)dt$$

$$\frac{1}{1000 + \frac{1}{20}y}dy = dt$$

$$\int \frac{1}{1000 + \frac{1}{20}y}dy = \int dt$$

$u = 1000 + \frac{1}{20}y$
$\frac{du}{dy} = \frac{1}{20}$
$du = \frac{1}{20}dy$
$20du = dy$

$$\int \frac{1}{u}(20du) = \int dt$$

$$20\ln|u| = t + c$$

$$20\ln\left|1000 + \frac{1}{20}y\right| = t + c$$

$$\ln\left|1000 + \frac{1}{20}y\right| = \frac{1}{20}t + C$$

$$e^{\frac{1}{20}t + c} = 1000 + \frac{1}{20}y$$

$$e^{\frac{1}{20}t} \cdot e^{c} = 1000 + \frac{1}{20}y$$

$$c \cdot e^{\frac{1}{20}t} = 1000 + \frac{1}{20}y$$

$$ce^{\frac{1}{20}t} - 1000 = \frac{1}{20}y$$

$$20ce^{\frac{1}{20}t} - 20{,}000 = y$$

$$y = 20ce^{\frac{1}{20}t} - 20{,}000$$

@ (0,0)

$$0 = 20ce^{\frac{1}{20}(0)} - 20{,}000$$

$$0 = 20ce^{0} - 20{,}000$$

$$20{,}000 = 20c$$

$$1000 = c$$

$$y = 20(1000)e^{\frac{1}{20}x} - 20{,}000$$

$$y = 20{,}000e^{\frac{1}{20}t} - 20{,}000$$

Advanced Integration
10.5B – Differential Equations Applications

Tastes Like Murder

#2) After marathon watching CSI Miami, George witnesses a crime scene from his cardboard home. George observes leftover pizza that has recently been discarded (murdered). After carefully outlining the pizza with chalk, he puts his CSI training to work. A pizza corpse cools at a rate proportional to the difference between its temperature and the temperature of the surrounding air. If $y(t)$ is the temperature in degrees Fahrenheit of the body t hours after the murder (meal), and if the air temperature is 70°, then $y' = -0.32(y - 70)$ and y(0) = 98.6degrees

t = hours after meal
y = temp °F of body
y' = net change in °F/hour

a. Solve this differential with given initial condition

$-.32 = -\frac{8}{25}$

$$\frac{dy}{dt} = -.32y + 22.4$$

$$dy = \left(-\frac{8}{25}y + 22.4\right)dt$$

$$\int \frac{1}{-\frac{8}{25}y + 22.4}\,dy = \int dt$$

$u = -\frac{8}{25}y + 22.4$

$\frac{du}{dy} = -\frac{8}{25}$

$du = -\frac{8}{25}dy$

$-\frac{25}{8}du = dy$

$$\int \frac{1}{u}\left(-\frac{25}{8}\right)du = \int dt$$

$$-\frac{25}{8}\ln|u| = t + C$$

$$-\frac{25}{8}\ln\left(-\frac{8}{25}y + 22.4\right) = t + C$$

$$\ln\left|\frac{-8}{25}y + 22.4\right| = \frac{8}{-25}t + C \quad \text{(Log form)}$$

$$e^{\frac{-8}{25}t + C} = \frac{-8}{25}y + 22.4 \quad \text{(Exp Form)}$$

$$e^{\frac{-8}{25}t}e^{C} = \frac{-8}{25}y + 22.4$$

$$Ce^{\frac{-8}{25}t} = \frac{-8}{25}y + 22.4$$

$$Ce^{-\frac{8}{25}t} - 22.4 = -\frac{8}{25}y$$

$$Ce^{-8/25\,t} + 70 = y$$

@ (0, 98.6)

$$\boxed{\begin{aligned} Ce^{-8/25(0)} + 70 &= 98.6 \\ Ce^{0} + 70 &= 98.6 \\ C &= 28.6 \\ y &= 28.6e^{-8/25\,t} + 70 \end{aligned}}$$

b. Use your answer to part (a) to estimate how long ago the pizza was eaten if the temperature of the pizza was 80° when George discovered it.

$$y = 28.6e^{\frac{-8}{25}t} + 70$$

$$80 = 28.6e^{-\frac{8}{25}t} + 70$$

$$10 = 28.6e^{-8/25\,t}$$

$$\frac{10}{28.6} = e^{\frac{-8}{25}t} \quad \text{(Exp Form)}$$

$$\ln\frac{10}{28.6} = \frac{-8}{25}t \quad \text{(Log form)}$$

$$\frac{25}{-8}\ln\frac{10}{28.6} = t$$

$$3.28 \approx t$$

The pizza was eaten 3.28 minutes ago.

Panhandling

#3) Via panhandling, George meets 30 new people each day, but each day he forgets 20% of all the people that he knows.

y(t) = total people George remembers

y' = net change in people/day

t = days after meeting George

If *y(t)* is the total people who he remembers after *t* days, then make a differential equation.

$$\frac{dy}{dt} = 30 - .20y$$

$$\frac{dy}{dt} = 30 - \frac{1}{5}y$$

Solve this differential equation subject to the initial condition that George knew no one at birth.

(0,0)

$$\frac{dy}{dt} = 30 - \frac{1}{5}y$$

$$dy = \left(30 - \frac{1}{5}y\right)dt$$

$$\int \frac{1}{30 - \frac{1}{5}y}\,dy = \int dt$$

$$u = 30 - \frac{1}{5}y$$
$$\frac{du}{dy} = -\frac{1}{5}$$
$$du = -\frac{1}{5}\,dy$$
$$-5du = dy$$

$$\int \frac{1}{u}(-5du) = t + C$$

$$-5\int \frac{1}{u}\,du = t + C$$

$$-5\ln|u| = t + C$$

$$-5\ln\left|30 - \frac{1}{5}y\right| = t + C$$

$$\ln\left|30 - \frac{1}{5}y\right| = -\frac{1}{5}t + C$$ (Log form)

$$e^{-\frac{1}{5}t + C} = 30 - \frac{1}{5}y$$ (Exp Form)

$$e^{-\frac{1}{5}t} \cdot e^{C} = 30 - \frac{1}{5}y$$

$$Ce^{-\frac{1}{5}t} = 30 - \frac{1}{5}y$$

$$Ce^{-\frac{1}{5}t} - 30 = -\frac{1}{5}y$$

$$Ce^{-\frac{1}{5}t} + 150 = y$$

@ (0,0)

$$Ce^{-\frac{1}{5}(0)} + 150 = 0$$

$$Ce^{0} + 150 = 0$$

$$C = -150$$

$$y = -150e^{-\frac{1}{5}t} + 150$$

Screams of Terror

#4) By way of George just being George, he sends 20 people running and screaming in terror away from him each day. But, as a defense mechanism 90% of those who run in terror block out the memory of every meeting George.

If $y(t)$ is the total people who remember George after t days, then make a differential equation.

$$\frac{dy}{dt} = +20 - .90y$$

$$\frac{dy}{dt} = 20 - \frac{9}{10}y$$

t = days
y = total people who remember George
y' = net people/day who remember George

Solve this differential equation subject to the initial condition that no one ran away from George in terror before he was born. $(0,0)$

$$\frac{dy}{dt} = 20 - \frac{9}{10}y$$

$$dy = \left(20 - \frac{9}{10}y\right)dt$$

$$\int \frac{1}{20 - \frac{9}{10}y}\,dy = \int dt$$

$u = 20 - \frac{9}{10}y$

$\frac{du}{dt} = \frac{-9}{10}$

$du = -\frac{9}{10}dt$

$\frac{10}{-9}du = dt$

$$\int \frac{1}{u}\left(\frac{10}{-9}du\right) = t + C$$

$$\frac{10}{-9}\int \frac{1}{u}\,du = t + C$$

$$\frac{10}{-9}\ln|u| = t + C$$

$$\frac{10}{-9}\ln\left|20 - \frac{9}{10}y\right| = t + C$$

$$\ln\left|20 - \frac{9}{10}y\right| = \frac{-9}{10}t + C$$

$$e^{-\frac{9}{10}t + C} = 20 - \frac{9}{10}y$$

$$e^{-\frac{9}{10}t} \cdot e^{C} = 20 - \frac{9}{10}y$$

$$Ce^{-\frac{9}{10}t} = 20 - \frac{9}{10}y$$

$$Ce^{-\frac{9}{10}t} - 20 = -\frac{9}{10}y$$

$$Ce^{-\frac{9}{10}t} + \frac{200}{9} = y$$

$$Ce^{-\frac{9}{10}(0)} + \frac{200}{9} = 0$$

$$Ce^{0} + \frac{200}{9} = 0$$

$$C = \frac{-200}{9}$$

$$y = \frac{-200}{9}e^{-\frac{9}{10}t} + \frac{200}{9}$$

#1) $y = 20{,}000e^{0.05t} - 20{,}000$

#2) a. $y = 28.6e^{-0.32t} + 70$
b. about 3.28 hours

#3) $y' = 30 - 0.2y$.
$y = 150 - 150e^{-0.2t}$

#4) ~~$y' = 20 - 0.8y$.~~
~~$y = 25 - 25e^{-0.8t}$~~