Basic Integration

10.6 – Review

Simplify verses The U

$$\int \frac{e^x}{3+e^x}\,dx = \int \frac{e^x}{u}\,\frac{1}{e^x}\,du$$

$$\boxed{\begin{aligned} u &= 3+e^x \\ \frac{du}{dx} &= e^x \\ du &= e^x\,dx \\ \frac{1}{e^x}\,du &= dx \end{aligned}}$$

$$= \int \frac{1}{u}\,du$$

$$= \ln|u| + C$$

$$= \ln|3+e^x| + C$$

$$\int (e^x + 3)\,dx = e^x + 3x + C$$

$$\int \frac{\ln(e^{2x})}{x^2}\,dx = \int \frac{2\cancel{x}\,\overset{=1}{\ln e}}{x^{\cancel{2}}}\,dx$$

$$= \int \frac{2}{x}\,dx$$

$$= 2\ln|x| + C$$

Trig Identities

Pythagorean

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x = 1 - \sin^2 x$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

Double Angle

$\sin(2x) = 2\sin x\cos x$

$$\begin{aligned} \cos(2x) &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1 \end{aligned}$$

Example:

$$\int \cos^3 x\,dx = \int \cos^2 x \cdot \cos x\,dx$$

$$= \int (1-\sin^2 x)\cos x\,dx$$

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x\,dx \\ \frac{du}{\cos x} &= dx \end{aligned}}$$

$$= \int (1-u^2)\cos x\,\frac{du}{\cos x}$$

$$= \int (1-u^2)\,du$$

$$= u - \frac{1}{3}u^3 + C$$

$$= \sin x - \frac{1}{3}\sin^2 x + C$$

Basic Integration

10.6 – Review

Evaluate the indefinite integrals using The U

$$\int \frac{x}{\sqrt{x+1}}dx = \int \frac{x}{\sqrt{u}}\,du$$

$$\boxed{\begin{aligned}u &= x+1\\ du &= dx\end{aligned}}$$

$u-1=x$

$$= \int \frac{u-1}{u^{\frac{1}{2}}}\,du$$

$$= \int\left(\frac{u}{u^{\frac{1}{2}}} - \frac{1}{u^{1/2}}\right)du$$

$$= \int\left(u^{1/2} - u^{-\frac{1}{2}}\right)du$$

$$= \frac{2}{3}u^{3/2} - 2u^{1/2} + C$$

$$= \frac{2}{3}\sqrt{(x+1)^3} - 2\sqrt{x+1} + C$$

Solve the differential equation.

$$\frac{dy}{dx} = (\sin x)y^2$$

$$dy = (\sin x)\,y^2\,dx$$

$$\int y^{-2}dy = \int \sin x\,dx$$

$$-1y^{-1} = -\cos x + C$$

$$\frac{-1}{y} = -\cos x + C$$

$$-1 = (-\cos x + C)\,y$$

$$\frac{-1}{-\cos x + C} = y$$

$$y = \frac{1}{\cos x + C}$$

Initial Value

Solve for y if $\frac{dy}{dx} = (xy)^2$ and $y = 1$ when $x = 1$

$$dy = x^2y^2\,dx$$

$$\int y^{-2}dy = \int x^2dx$$

$$-y^{-1} = \frac{1}{3}x^3 + C$$

$$\frac{-1}{y} = \frac{1}{3}x^3 + C$$

$$-1 = \left(\frac{1}{3}x^3 + C\right)y$$

$$\frac{-1}{\frac{1}{3}x^3 + C} = y$$

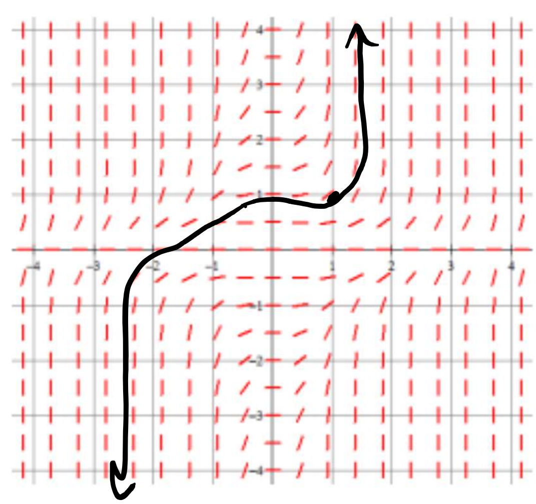
@ (1,1) $\quad 1 = \dfrac{-1}{\frac{1}{3}(1)^3 + C}$

$$1 = \frac{-1}{\frac{1}{3} + C}$$

$$\frac{1}{3} + C = -1$$

$$C = -4/3$$

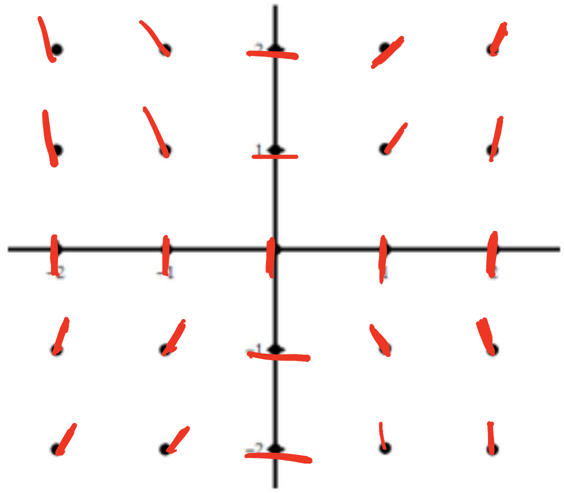
$$y = \frac{-1}{\frac{1}{3}x^3 - 2/3}$$



Use the differential to answer the following.

$$\frac{dy}{dx} = \frac{2x}{y}$$

A) Fill in the slope field.



B) Write the equation of the line tangent to the solution curve at point (2, 1)

Point: $(2,1)$

Slope: $\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{2(2)}{1} = 4$

Point Slope:

$y - y_1 = m(x - x_1)$

$y - (1) = 4(x - (2))$

$y - 1 = 4(x - 2)$

C) Find the particular solution with initial condition of $f(2) = 1$

$\frac{dy}{dx} = \frac{2x}{y}$

$\int y\,dy = \int 2x\,dx$

$\frac{1}{2}y^2 = x^2 + C$

$y^2 = 2x^2 + C$

$y = \pm\sqrt{2x^2 + C}$

$(2,1)$

$y = \sqrt{2x^2 + C}$

$1 = \sqrt{2(2)^2 + C}$

$1 = \sqrt{8 + C}$

$1 = 8 + C$

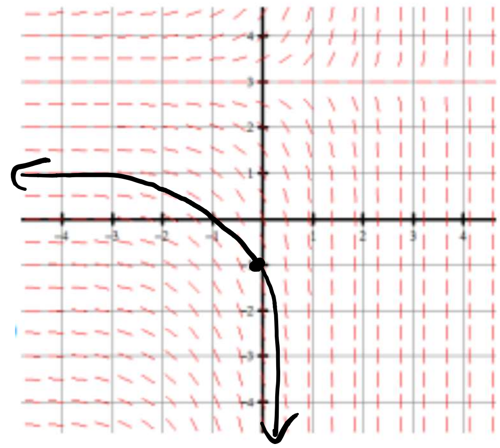
$-7 = C$

$y = \sqrt{2x^2 - 7}$

Solve the differential equation.

$$\frac{dy}{dx} = (y+2)e^x$$

A) Sketch a particular solution through the point (0, -1)



$\frac{dy}{dx} = (y+2)e^x$

$\boxed{u = y+2 \quad du = dy}$

$\int \frac{1}{y+2}\,dy = \int e^x\,dx$

$\int \frac{1}{u}\,du = e^x + C$

$\ln|u| = e^x + C$

$\ln|y+2| = e^x + C$

$e^{e^x + C} = y + 2$

B) Find the particular solution with initial condition (0, -1)

$e^{e^x} \cdot e^C = y + 2$

$Ce^{e^x} - 2 = y$

$Ce^{e^{(0)}} - 2 = -1$

$Ce = 1$

$C = \frac{1}{e}$

$y = \frac{1}{e}e^{e^x} - 2$