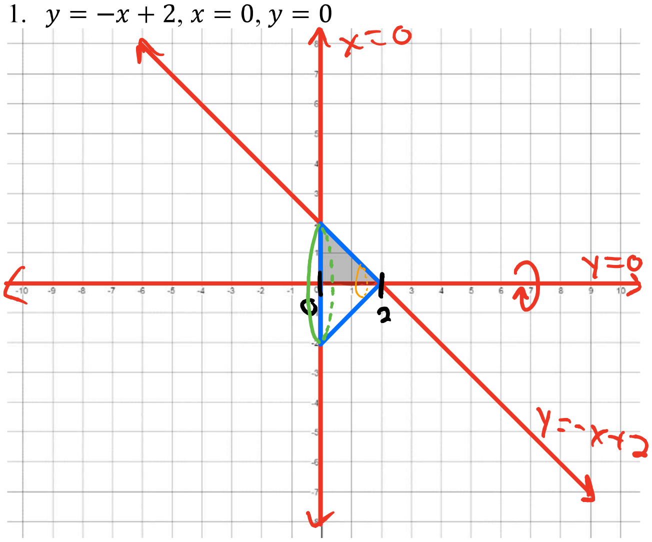
Sketch the area bounded by the equations and revolve it around the x-axis. Find the volume of the resulting solid. Leave the answers in terms of π.

1. $y = -x + 2,\ x = 0,\ y = 0$



② $R(x) = -x+2$

$R^2 = x^2 - 4x + 4$

③ D: [0,2]

④ $V = \pi \int_a^b R^2(x)\,dx$

$V = \pi \int_0^2 [4 - 4x + x^2]\,dx$

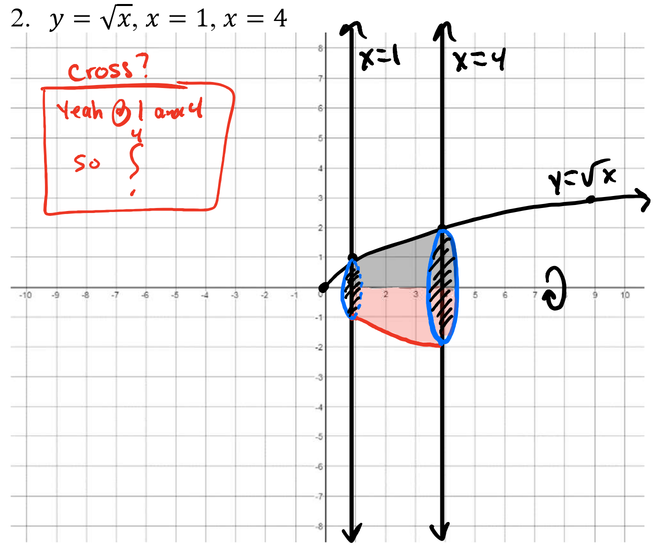
$V = \pi \left[4x - 2x^2 + \frac{1}{3}x^3\right]\Big|_0^2$

$V = \pi\left[4(2) - 2(2)^2 + \frac{1}{3}(2)^3\right] - \pi\left[4(0) - 2(0)^2 + \frac{1}{3}(0)^2\right]$

$V = \pi\left[8 - 8 + \frac{1}{3}(8)\right] - \pi[0]$

$V = \frac{8}{3}\pi\ un^3$

2. $y = \sqrt{x},\ x = 1,\ x = 4$



② $R(x) = \sqrt{x}$

$R^2 = x$

③ D: [1,4]

④ $V = \pi \int_a^b R^2(x)\,dx$

$V = \pi \int_1^4 (x)\,dx$

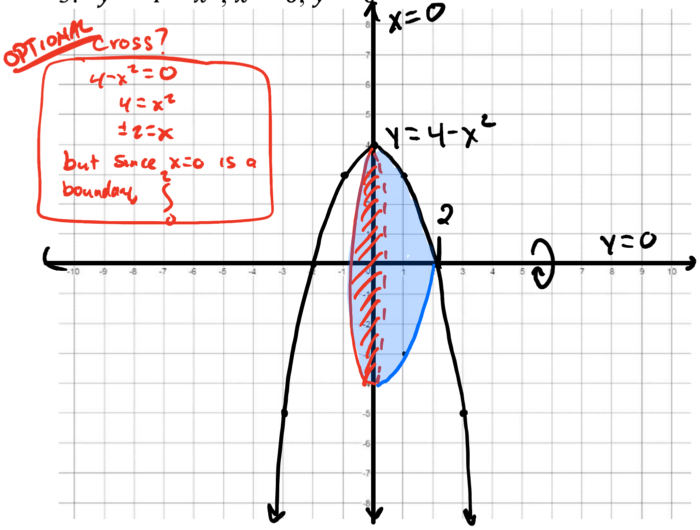
$V = \pi \frac{1}{2}x^2\Big|_1^4$

$V = \left[\pi \frac{1}{2}(4)^2\right] - \left[\pi \frac{1}{2}(1)^2\right]$

$V = \left[\pi\left(\frac{1}{2}\right)16\right] - \left[\pi \frac{1}{2}\right]$

$V = \frac{15}{2}\pi\ un^3$

3. $y = 4 - x^2$, $x = 0$, $y = 0$



② $R(x) = 4 - x^2$

$R^2 = 16 - 8x^2 + x^4$

③ $D: [0, 2]$

④ $V = \pi \int_a^b R^2(x)\,dx$

$V = \pi \int_0^2 [16 - 8x^2 + x^4]$

$V = \pi \left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]\Big|_0^2$

$V = \pi\left[16(2) - \frac{8}{3}(2)^3 + \frac{1}{5}(2)^5\right] - \pi\left[16(0) - \frac{8}{3}(0)^3 + \frac{1}{5}(0)^5\right]$

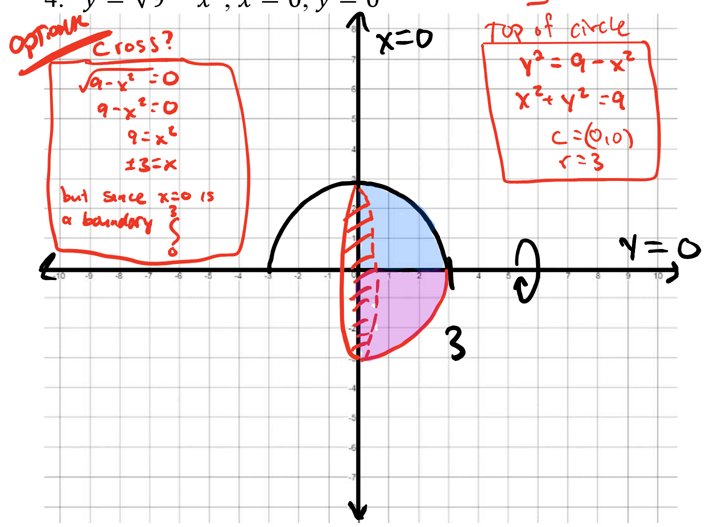
$V = \pi\left[32 - \frac{64}{3} + \frac{32}{5}\right] - \pi[0]$

$V = \pi\left[\frac{480}{15} - \frac{320}{15} + \frac{96}{15}\right]$

$V = \pi\left[\frac{256}{15}\right]$

$V = \frac{256}{15}\pi$ un^3

4. $y = \sqrt{9 - x^2}$, $x = 0$, $y = 0$



② $R(x) = \sqrt{9 - x^2}$

$R^2 = 9 - x^2$

③ $D = [0, 3]$

④ $V = \pi \int_a^b R^2(x)\,dx$

$V = \pi \int_0^3 [9 - x^2]\,dx$

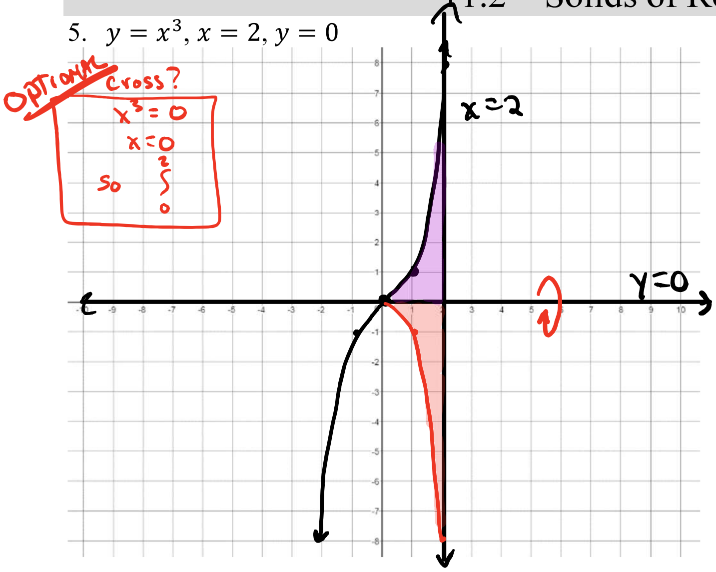
$V = \pi\left[9x - \frac{1}{3}x^3\right]\Big|_0^3$

$V = \pi\left[9(3) - \frac{1}{3}(3)^3\right] - \pi\left[9(0) - \frac{1}{3}(0)^3\right]$

$V = \pi[27 - 9] - \pi[0]$

$V = 18\pi$ un^3

5. $y = x^3, x = 2, y = 0$



② $R(x) = x^3$

$R^2 = x^6$

③ $D = [0, 2]$

④ $V = \pi \int_a^b R^2(x)\,dx$

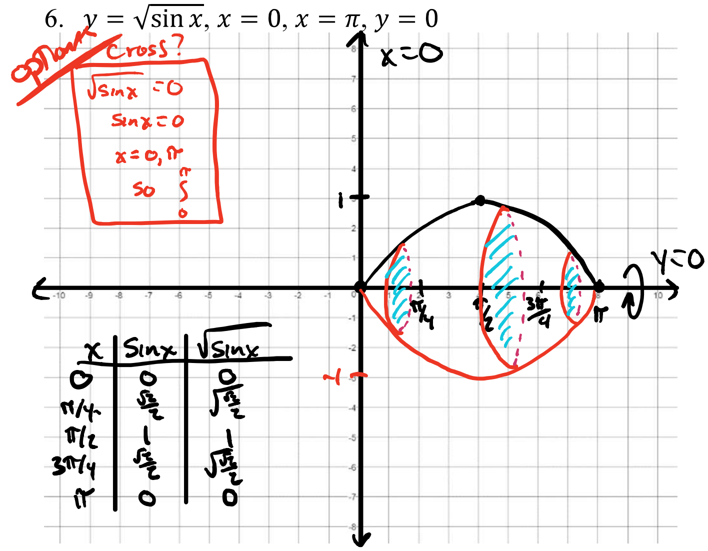
$V = \pi \int_0^2 x^6\,dx$

$V = \pi \frac{1}{7}x^7 \Big|_0^2$

$V = \left[\pi \frac{1}{7}(2)^7\right] - \left[\pi \frac{1}{7}(0)^7\right]$

$V = \frac{128}{7}\pi \text{ un}^3$

6. $y = \sqrt{\sin x}, x = 0, x = \pi, y = 0$



x	Sinx	√Sinx
0	0	0
π/4	√2/2	√(√2/2)
π/2	1	1
3π/4	√2/2	√(√2/2)
π	0	0

② $R(x) = \sqrt{\sin x}$

$R^2 = \sin x$

③ $D = [0, \pi]$

④ $V = \pi \int_a^b R^2(x)\,dx$

$V = \pi \int_0^\pi \sin x\,dx$

$V = \pi(-\cos x)\Big|_0^\pi$

$V = [-\pi\cos\pi] - [-\pi\cos 0]$

$V = [-\pi(-1)] - [-\pi(1)]$

$V = \pi + \pi$

$V = 2\pi \text{ un}^3$

Sketch the area bounded by the equations and revolve it around the y-axis. Find the volume of the resulting solid. Leave the answers in terms of π.

7. $y = -x + 2,\ x = 0,\ y = 0$

$y-2=-x$

$2-y=x$

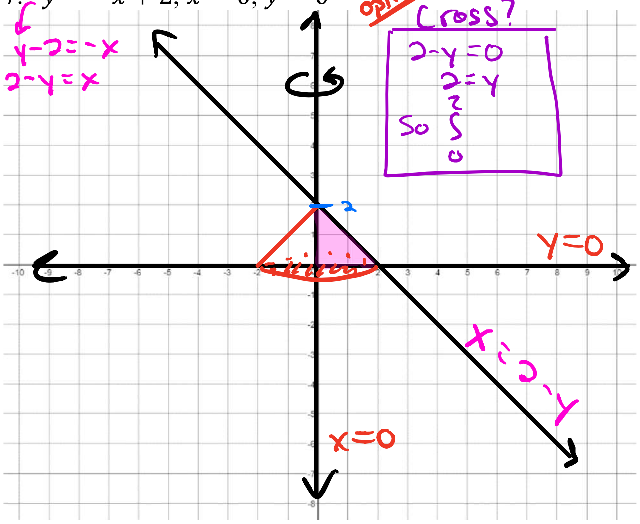
OPTIONAL

Cross?

$2-y=0$

$2=y$

So $\int_0^2$



② $R(y) = -y+2$

$R^2 = y^2 - 4y + 4$

③ $R: [0,2]$

④ $V = \pi \int_a^b R^2(y)\,dy$

$V = \pi \int_0^2 [4 - 4y + y^2]\,dy$

$V = \pi \left[4y - 2y^2 + \frac{1}{3}y^3\right]\Big|_0^2$

$V = \pi\left[4(2) - 2(2)^2 + \frac{1}{3}(2)^3\right] - \pi\left[4(0) - 2(0)^2 + \frac{1}{3}(0)^2\right]$

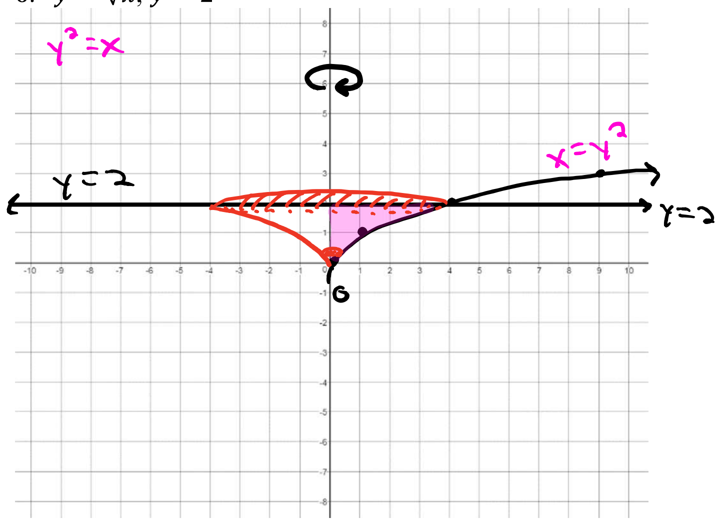
$V = \pi\left[8 - 8 + \frac{1}{3}(8)\right] - \pi[0]$

$V = \frac{8}{3}\pi\ un^3$

Compare to # 1

8. $y = \sqrt{x},\ y = 2$

$y^2 = x$



② $R(y) = y^2$

$R^2 = y^4$

③ $R: [0,2]$

④ $V = \pi \int_a^b R^2(y)\,dy$

$V = \pi \int_0^2 y^4\,dy$

$V = \pi \frac{1}{5}y^5 \Big|_0^2$

$V = \left[\pi \frac{1}{5}(2)^5\right] - \left[\pi \frac{1}{5}(0)^5\right]$

$V = \frac{32}{5}\pi\ un^3$

Compare to #2

9. $y = 4 - x^2$, $x = 0$, $y = 0$

$y - 4 = -x^2$

$4 - y = x^2$

$\pm\sqrt{4-y} = x$

only need + b/c its revolved

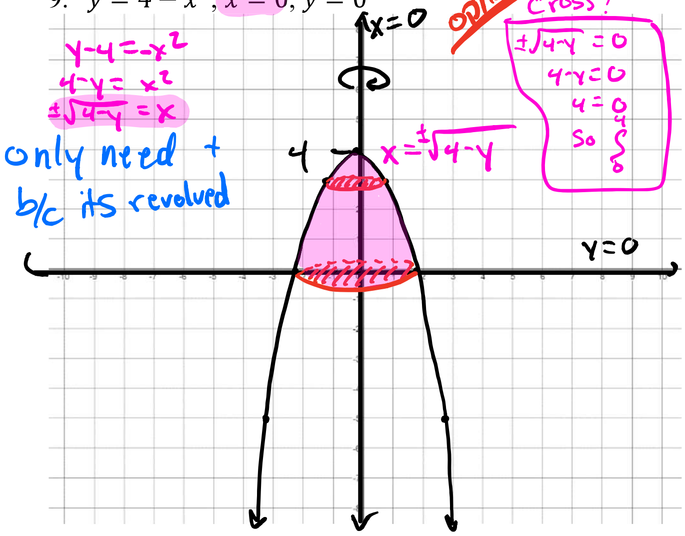
OPTIONAL: Cross?

$\pm\sqrt{4-y} = 0$

$4 - y = 0$

$4 = y$

So $\int_0^4$



② $R(y) = \sqrt{4-y}$

$R^2 = 4 - y$

③ $R: [0,4]$

④ $V = \pi \int_a^b R^2(y)\, dy$

$V = \pi \int_0^4 (4-y)\, dy$

$V = \pi \left(4y - \frac{1}{2}y^2\right)\Big|_0^4$

$V = \pi\left[4(4) - \frac{1}{2}(4)^2\right] - \pi\left[4(0) - \frac{1}{2}(0)^2\right]$

$V = \pi\left[16 - \frac{1}{2}(16)\right] - \pi[0]$

$V = \pi[16 - 8]$

$V = 8\pi \text{ un}^3$

Compare to #3

10. $y = \sqrt{9 - x^2}$, $x = 0$, $y = 0$

$y^2 = 9 - x^2$

$y^2 - 9 = -x^2$

$9 - y^2 = x^2$

$\pm\sqrt{9-y^2} = x$

only need + b/c its revolved

OPTIONAL: Cross?

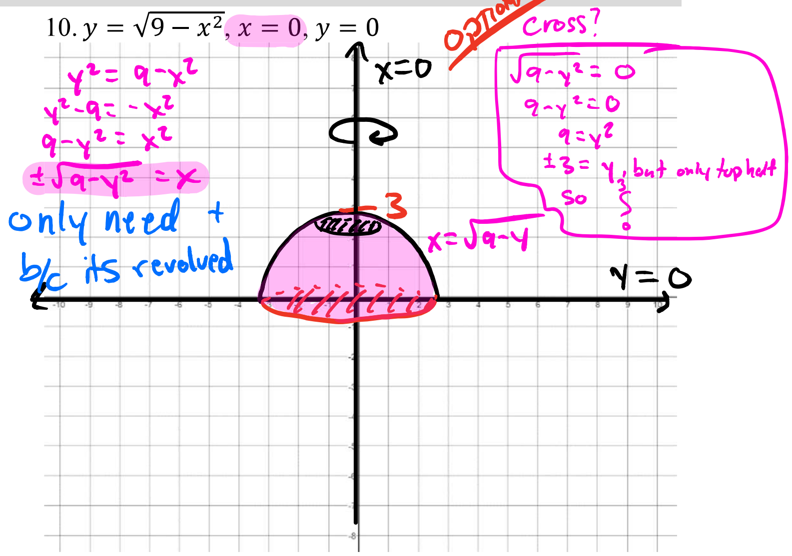
$\sqrt{9-y^2} = 0$

$9 - y^2 = 0$

$9 = y^2$

$\pm 3 = y$, but only top half

So $\int_0^3$



② $R(y) = \sqrt{9-y^2}$

$R^2 = 9 - y^2$

③ $R: [0,3]$

④ $V = \pi \int_a^b R^2(y)\, dy$

$V = \pi \int_0^3 (9-y^2)\, dy$

$V = \pi \left[9y - \frac{1}{3}y^3\right]\Big|_0^3$

$V = \pi\left[9(3) - \frac{1}{3}(3)^3\right] - \pi\left[9(0) - \frac{1}{3}(0)^3\right]$

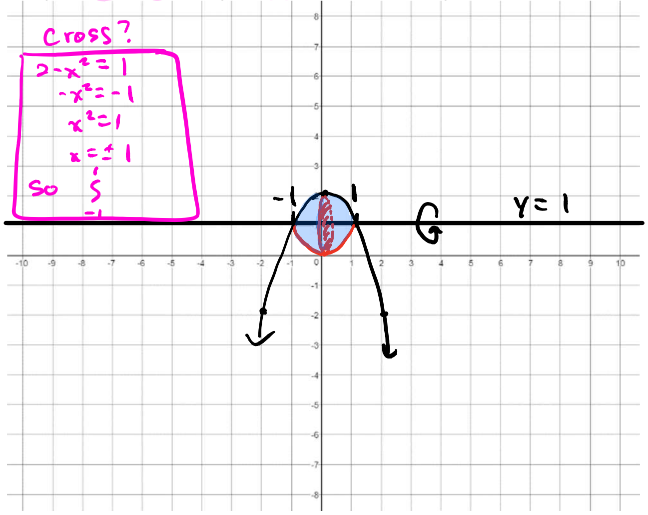
$V = \pi[27 - 9] - \pi[0]$

$V = 18\pi \text{ un}^3$

Compare to #4

Sketch the area bounded by the equations and revolve it around the given line. Find the volume of the resulting solid. Leave the answers in terms of π.

11. $y = 2 - x^2$ and $y = 1$ about the line $y = 1$



② $R(x) = \text{Top} - \text{Bottom} = (2-x^2) - (1) = 1 - x^2$

$R^2 = 1 - 2x^2 + x^4$

③ $D: [-1, 1]$

④ $V = \pi \int_a^b [R(x)]^2 dx$

$V = \pi \int_{-1}^{1} [1 - 2x^2 + x^4] dx$

$V = \pi \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right]\Big|_{-1}^{1}$

$V = \pi\left[(1) - \frac{2}{3}(1)^3 + \frac{1}{5}(1)^5\right] - \pi\left[(-1) - \frac{2}{3}(-1)^3 + \frac{1}{5}(-1)^5\right]$

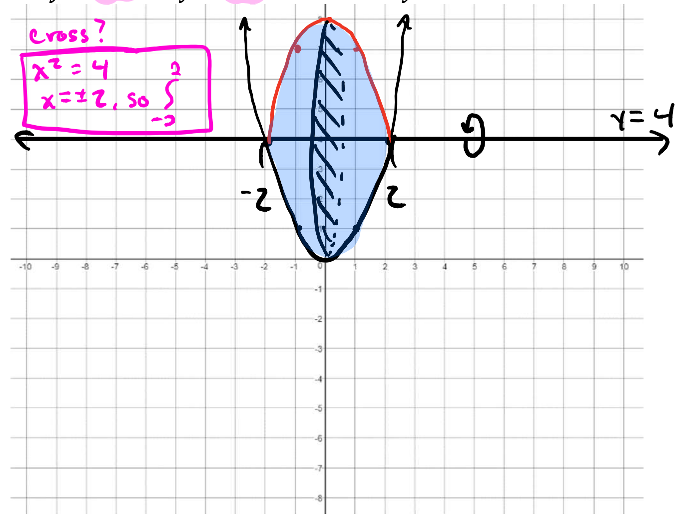
$V = \pi\left[1 - \frac{2}{3} + \frac{1}{5}\right] - \pi\left[-1 + \frac{2}{3} - \frac{1}{5}\right]$

$V = \pi\left[2 - \frac{4}{3} + \frac{2}{5}\right]$

$V = \pi\left[\frac{30}{15} - \frac{20}{15} + \frac{6}{15}\right]$

$V = \frac{16}{15}\pi \text{ un}^3$

12. $y = x^2$ and $y = 4$ about the line $y = 4$



② $R(x) = \text{Top} - \text{Bottom} = (4) - (x^2) = 4 - x^2$

$R^2 = 16 - 8x^2 + x^4$

③ $D: [-2, 2]$

④ $V = \pi \int_a^b R^2(x)\, dx$

$V = \pi \int_{-2}^{2} [16 - 8x^2 + x^4] dx$

$V = \pi\left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]\Big|_{-2}^{2}$

$V = \pi\left[16(2) - \frac{8}{3}(2)^3 + \frac{1}{5}(2)^5\right] - \pi\left[16(-2) - \frac{8}{3}(-2)^3 + \frac{1}{5}(-2)^5\right]$

$V = \pi\left[32 - \frac{64}{3} + \frac{32}{5}\right] - \pi\left[-32 + \frac{64}{3} - \frac{32}{5}\right]$

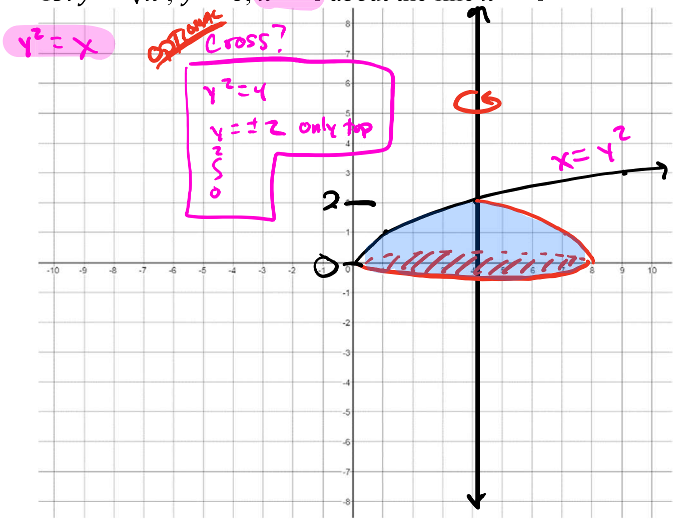
$V = \pi\left[64 - \frac{128}{3} + \frac{64}{5}\right]$

$V = \pi\left[\frac{960}{15} - \frac{640}{15} + \frac{192}{15}\right]$

$V = \frac{512}{15}\pi \text{ un}^3$

13. $y = \sqrt{x}$, $y = 0$, $x = 4$ about the line $x = 4$

$y^2 = x$



② $R(y) = \text{RIGHT} - \text{LEFT} = (4) - (y^2) = 4 - y^2$

$R^2 = 16 - 8y^2 + y^4$

③ $R: [0, 2]$

④ $V = \pi \int_a^b R^2(y)\, dy$

$V = \pi \int_0^2 [16 - 8y^2 + y^4]\, dy$

$V = \pi \left[16y - \frac{8}{3}y^3 + \frac{1}{5}y^5\right]\Big|_0^2$

$V = \pi\left[16(2) - \frac{8}{3}(2)^3 + \frac{1}{5}(2)^5\right] - \pi\left[16(0) - \frac{8}{3}(0)^3 + \frac{1}{5}(0)^5\right]$

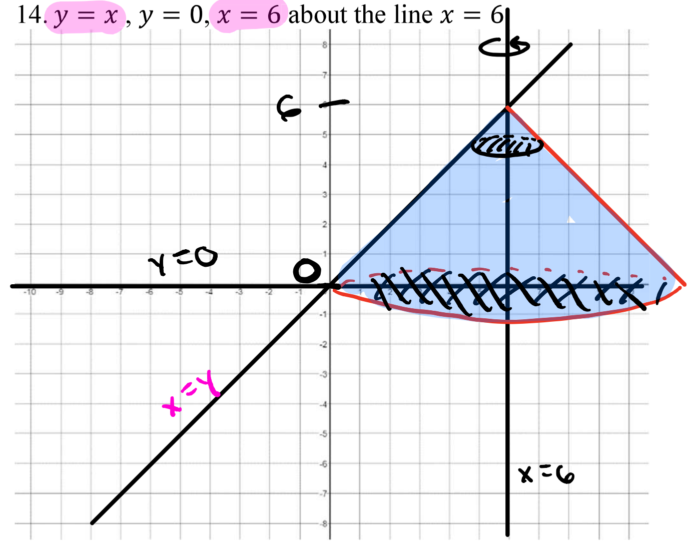
$V = \pi\left[32 - \frac{64}{3} + \frac{32}{5}\right] - \pi[0]$

$V = \pi\left[\frac{480}{15} - \frac{320}{15} + \frac{96}{15}\right]$

$V = \frac{256}{15}\pi \text{ un}^3$

Compare to #12

14. $y = x$, $y = 0$, $x = 6$ about the line $x = 6$



② $R(y) = \text{RIGHT} - \text{LEFT} = (6) - (y) = 6 - y$

$R^2 = 36 - 12y + y^2$

③ $R: [0, 6]$

④ $V = \pi \int_a^b R^2(y)\, dy$

$V = \pi \int_0^6 [36 - 12y + y^2]\, dy$

$V = \pi \left[36y - 6y^2 + \frac{1}{3}y^3\right]\Big|_0^6$

$V = \pi\left[36(6) - 6(6)^2 + \frac{1}{3}(6)^3\right] - \pi\left[36(0) - 6(0)^2 + \frac{1}{3}(0)^3\right]$

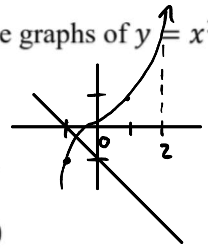
$V = \pi\left[\cancel{36(6)} - \cancel{6(36)} + \frac{1}{3}(216)\right] - \pi[0]$

$V = 72\pi \text{ un}^3$

11.2 Solids of Revolution (Discs)

Test Prep

1. What is the area of the region between the graphs of $y = x^3$ and $y = -x - 1$ from $x = 0$ to $x = 2$?



$$A = \int_0^2 [(x^3) - (-x-1)]\,dx$$
$$A = \int_0^2 [x^3 + x + 1]\,dx$$
$$A = \left[\tfrac{1}{4}x^4 + \tfrac{1}{2}x^2 + x\right]_0^2$$
$$A = \left[\tfrac{1}{4}(2)^4 + \tfrac{1}{2}(2)^2 + (2)\right] - [0]$$
$$A = \tfrac{1}{4}(16) + \tfrac{1}{2}(4) + 2$$
$$A = 4 + 2 + 2$$
$$A = 8$$

(A) 0 (B) 4 (C) 5
(D) 8 (E) 10

2. Let $F(x)$ be an antiderivative of $\frac{2(\ln x)^4}{3x}$. If $F(2) = 0$, then $F(8) =$

$u = \frac{1}{3x}$
$du = \frac{1}{3}\ln x\,dx$

(A) 5.163 (B) 0.860 (C) 0.184
(D) 0.180 (E) 0.004

3. The average value of $f(x) = -\sin x$ on the interval $[-2, 4]$ is

cos x is an even function so $\cos(-x) = \cos x$

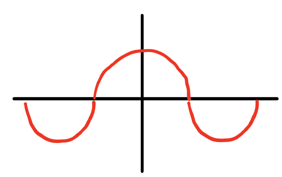
$$Av = \frac{1}{4-(-2)}\int_{-2}^{4} -\sin x\,dx = \frac{1}{6}\int_{-2}^{4} -\sin x\,dx = \frac{1}{6}\cos x\Big|_{-2}^{4} = \left[\tfrac{1}{6}\cos 4\right] - \left[\tfrac{1}{6}\cos(-2)\right]$$
$$= \tfrac{1}{6}\cos 4 - \tfrac{1}{6}\cos(-2)$$
$$= \tfrac{1}{6}\cos 4 - \tfrac{1}{6}\cos 2$$

(A) $\frac{\cos 4 + \cos 2}{6}$ (B) $\frac{\cos 2 - \cos 4}{2}$ (C) $\frac{\cos 4 + \cos 2}{2}$

(D) $\frac{\cos 4 - \cos 2}{2}$ (E) $\frac{\cos 4 - \cos 2}{6}$

Even

$y = \cos x$



4. If $y = \int_1^{x^2} \sqrt{t^2 + 3}\,dt$, then $F'(2) =$

2nd FTC

Chain Rule!

$F'(x) = 2x\sqrt{(x^2)^2 + 3}$
$F'(x) = 2x\sqrt{x^4 + 3}$

$F'(2) = 2(2)\sqrt{(2)^4 + 3}$
$= 4\sqrt{16 + 3}$
$= 4\sqrt{19}$

(A) $4\sqrt{19}$ (B) $2\sqrt{19}$ (C) $4\sqrt{7}$
(D) $2\sqrt{7}$ (E) $\sqrt{7}$