Area and Volume

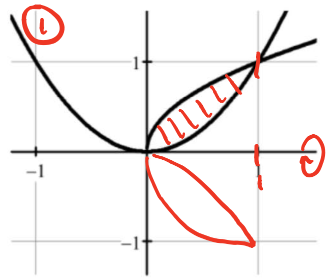
11.3 – Solids of Revolution (Washer Method)

Volume of a Solid of Revolution Washers

$$V = \pi \int_a^b [R^2(x) - r^2(x)]dx$$

Where R(x) is the radius of the outer function and r(x) is the radius of the inner function.

1. Find the volume if the region enclosed by $y = \sqrt{x}$ and $y = x^2$ is rotated about the x-axis.



② $R(x) = \sqrt{x}$
$r(x) = x^2$

③ $D: [0,1]$

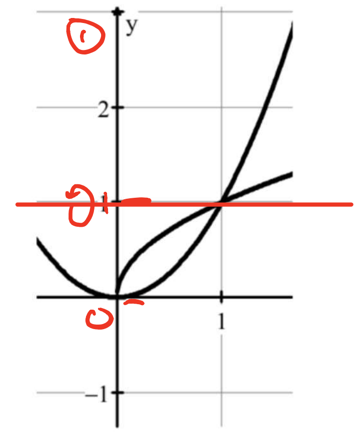
④ $V = \pi \int_a^b [R^2 - r^2] dx$

$V = \pi \int_a^b [R^2(x) - r^2(x)] dx$

$V = \pi \int_0^1 [(\sqrt{x})^2 - (x^2)^2] dx$

$V = \pi \int_0^1 [x - x^4] dx$

2. Find the volume if the region enclosed by $y = \sqrt{x}$ and $y = x^2$ is rotated about the line $y = 1$.



② $R(x) = 1 - x^2$
$r(x) = 1 - \sqrt{x}$

③ $R: [0,1]$

④ $V = \pi \int_a^b [R^2 - r^2] dx$

$V = \pi \int_0^1 [(1 - x^2)^2 - (1 - \sqrt{x})^2] d$

or

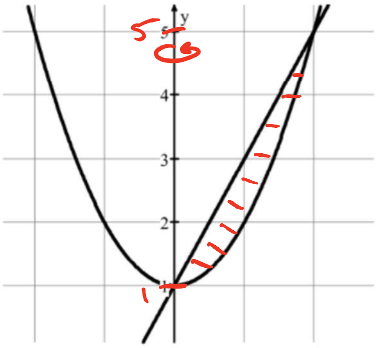
11.3 – Solids of Revolution (Washer Method)

3. Find the volume if the region enclosed by $y = x^2 + 1$ and $y = 2x + 1$ is rotated about the y-axis

$y = x^2 + 1$ | $y = 2x + 1$

$y - 1 = x^2$ | $y - 1 = 2x$

$\pm\sqrt{y-1} = x$ | $\frac{1}{2}y - \frac{1}{2} = x$



② $R(x) = \sqrt{y-1}$

$r(x) = \frac{1}{2}y - \frac{1}{2}$

③ $R: [1, 5]$

④ $V = \pi \int_a^b [R^2 - r^2]\,dx$

$V = \pi \int_1^5 \left[(\sqrt{y-1})^2 - (\tfrac{1}{2}y - \tfrac{1}{2})^2\right] dy$

$V = \pi \int_1^5 \left[(y-1) - (\tfrac{1}{4}y^2 - \tfrac{1}{2}y + \tfrac{1}{4})\right] dy$

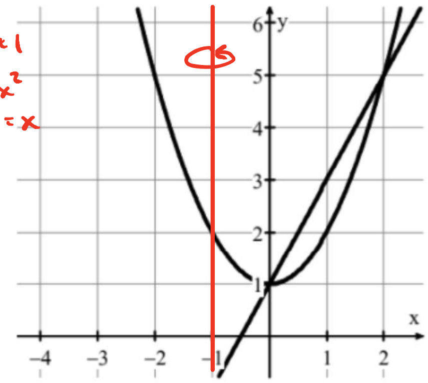
$V = \pi \int_1^5 \left[\tfrac{1}{2}y - \tfrac{5}{4}y - \tfrac{1}{4}y^2 - \tfrac{1}{2}y\right] dy$

4. Find the volume if the region enclosed by $y = x^2 + 1$ and $y = 2x + 1$ is rotated about the line $x = -1$.

$y = 2x + 1$ | $y = x^2 + 1$

$y - 1 = 2x$ | $y - 1 = x^2$

$\frac{1}{2}y - \frac{1}{2} = x$ | $\pm\sqrt{y-1} = x$



② $R(x) = (\sqrt{y-1}) - (-1) = \sqrt{y-1} + 1$

$r(x) = (\tfrac{1}{2}y - \tfrac{1}{2}) - (-1) = \tfrac{1}{2}y + \tfrac{1}{2}$

③ $R: [1, 5]$

④ $V = \pi \int_a^b [R^2 - r^2]\,dx$

$V = \pi \int_1^5 \left[(\sqrt{y-1} + 1)^2 - (\tfrac{1}{2}y + \tfrac{1}{2})^2\right] dy$