Area and Volume

11.4 – Perpendicular Cross Sections

Volume of a Solid with known Cross Sections

$$V = \int_a^b A(x)dx$$

Where ***A*** is the ***area*** of a cross section perpendicular to the x-axis.

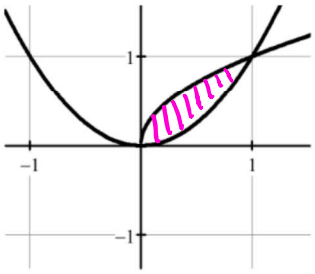
$$V = \int_a^b A(y)dy$$

Where ***A*** is the ***area*** of a cross section perpendicular to the y-axis.

$A_{square} = s^2$ $\qquad$ $A_{isosceles\ right\ triangle} = \frac{1}{2}s^2$

$A_{semicircle} = \frac{1}{2}\pi r^2$ $\qquad$ $A_{equilateral\ triangle} = \frac{\sqrt{3}}{4}s^2$

1. Find the volume of the solid whose base is bounded by $y = \sqrt{x}$ and $y = x^2$, with the indicated cross section taken perpendicular to the x-axis.



a. Square

$A(x) = s^2$

$A(x) = [\sqrt{x} - x^2]^2$

$V = \int_a^b A(x)dx$

$V = \int_0^1 [\sqrt{x} - x^2]^2 dx$

b. Equilateral triangle

$A(x) = \frac{\sqrt{3}}{4}s^2$

$A(x) = \frac{\sqrt{3}}{4}[\sqrt{x} - x^2]^2$

$V = \int_a^b A(x)dx$

$V = \frac{\sqrt{3}}{4}\int_0^1 [\sqrt{x} - x^2]^2 dx$

c. Semicircle

$A(x) = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi\left[\frac{\sqrt{x}-x^2}{2}\right]^2$

$= \frac{1}{2}\pi\frac{[\sqrt{x}-x^2]^2}{4}$

$A(x) = \frac{1}{8}\pi[\sqrt{x} - x^2]^2$

$V = \int_a^b A(x)dx$

$V = \frac{1}{8}\pi\int_0^1 [\sqrt{x} - x^2]^2 dx$

d. Isosceles right triangle (side is a base)

$A(x) = \frac{1}{2}s^2$

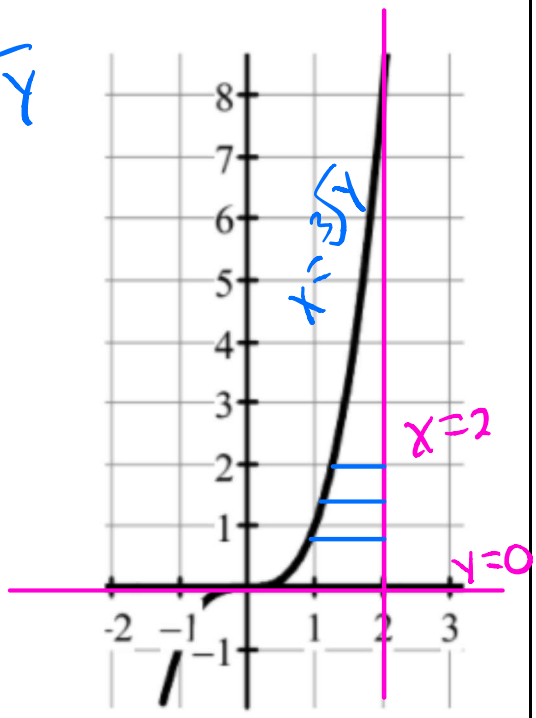
$A(x) = \frac{1}{2}[\sqrt{x} - x^2]^2$

$V = \int_a^b A(x)dx$

$V = \frac{1}{2}\int_0^1 [\sqrt{x} - x^2]^2 dx$

2. Find the volume of the solid whose base is bounded by $y = x^3,\ x = 2, y = 0$, with cross sections taken perpendicular to the y-axis that form a square.

$x = \sqrt[3]{y}$



$A(x) = s^2$

$A(x) = (2 - \sqrt[3]{y})^2$

$V = \int_a^b A(x)\,dx$

$V = \int_0^8 (2 - \sqrt[3]{y})^2\,dy$

BTW

$[\sqrt{x} - x^2]^2$

$= [x^{\frac{1}{2}} - x^2][x^{\frac{1}{2}} - x^2]$

$= x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} - x^{2 \cdot \frac{1}{2}} \cdot x^{\frac{1}{2}} - x^{2 \cdot \frac{1}{2}} \cdot x^{\frac{1}{2}} + x^2 \cdot x^2$

$= x^1 - x^{5/2} - x^{5/2} + x^4$

$= x - 2x^{5/2} + x^4$

$(2 - y^{\frac{1}{3}})^2 = 4 - 2y^{\frac{1}{3}} - 2y^{\frac{1}{3}} + y^{\frac{1}{3}} \cdot y^{\frac{1}{3}}$

$= 4 - 4y^{1/3} + y^{2/3}$