Name______________________________Date__________________Class__________

Homework 3.1

1. If $f(x)=\sqrt[3]{x}\left(x^2+4\right)$, find $f'(x)$ by first rewriting $f(x)$ in a polynomial form. Then, apply the power rule. $f(x)= x^{1/3}(x^2+4) = x^{1/3}(x^{6/3}+4)$ $f(x)= x^{7/3}+4x^{1/3}$ $\frac{d}{dx}f(x) = \frac{7}{3}x^{4/3}+\frac{4}{3}x^{-2/3}$ $= \frac{7x^{4/3}\cdot x^{2/3}}{3\cdot x^{2/3}}+\frac{4}{3x^{2/3}}$ $= \frac{7x^{6/3}+4}{3x^{2/3}}$ $\frac{df}{dx} = \frac{7x^2+4}{3\sqrt[3]{x^2}}$	2. If $f(x)=\sqrt[3]{x}\left(x^2+4\right)$, find $f'(x)$ by applying the product rule to find. $f(x)= x^{1/3}(x^2+4)$ $f'(x)= (x^{1/3})'(x^2+4)+(x^{1/3})(x^2+4)'$ $= \frac{1}{3}x^{-2/3}(x^2+4)+x^{1/3}(2x)$ $= \frac{x^2+4}{3x^{2/3}}+\frac{2x^{4/3}}{1}\cdot\frac{3x^{2/3}}{3x^{2/3}}$ $= \frac{x^2+4+6x^{6/3}}{3x^{2/3}}$ $= \frac{x^2+4+6x^2}{3\sqrt[3]{x^2}}$ $f'(x) = \frac{7x^2+4}{3\sqrt[3]{x^2}}$

For exercises 3 – 5, find the derivative of each function. Leave your answers as single rational expressions with no rational exponents, when necessary.

3. $f(x)=\left(x^2+2\right)\left(x^2-2x\right)$ $f(x)= x^4-2x^3+2x^2-4x$ Leibnitz $\frac{df}{dx} = 4x^3-6x^2+4-4$	4. $f(x)=\left(x^3-3x\right)\left(2x^2+3x+5\right)$ $f(x)= 2x^5+3x^4+5x^3-6x^3-9x^2-15x$ $f(x)= 2x^5+3x^4-x^3-9x^2-15x$ $\dot{f}(x)= 10x^4+12x^3-3x^2-18x-15$ Newton
5. $g(x)=\sqrt{x}\sin x$ $g(x)= x^{\frac{1}{2}}\sin x$ Lagrange $g'(x) = \frac{1}{2}x^{-\frac{1}{2}}\sin(x)+x^{\frac{1}{2}}\cos(x)$ $g'(x) = \frac{1}{2\sqrt{x}}\sin(x)+\sqrt{x}\cos(x)\cdot\frac{2\sqrt{x}}{2\sqrt{x}}$ $g'(x) = \frac{\sin(x)+2x\cos(x)}{2\sqrt{x}}$	6. $h(x)=\sin x\cos x$ $\dot{h}(x) = \cos(x)\cdot\cos(x)+\sin(x)[-\sin(x)]$ $\dot{h}(x) = \cos^2(x)-\sin^2(x)$

Find the slope of the normal line drawn to the graph of each function at the indicated value of x.

7. $g(x)=\sqrt{x}\sin x$ when $x=\pi$ From #5) $g'(x)=\frac{\sin(x)+2x\cos(x)}{2\sqrt{x}}$ $g'(\pi)=\frac{\sin(\pi)+2\pi\cos(\pi)}{2\sqrt{\pi}}$ $=\frac{0+2\pi(-1)}{2\sqrt{\pi}}$ $=\frac{-2\pi}{2\sqrt{\pi}}\cdot\frac{\sqrt{\pi}}{\sqrt{\pi}}$ $=\frac{-2\pi\cdot\sqrt{\pi}}{2\pi}$ $g'(\pi)=-\sqrt{\pi}$ SoN: $\frac{1}{\sqrt{\pi}}$	8. $h(x)=\sin x\cos x$ when $x=\frac{\pi}{4}$ From #6) $h'=\cos^2(x)-\sin^2(x)$ $h'(\frac{\pi}{4})=\cos^2(\frac{\pi}{4})-\sin^2(\frac{\pi}{4})$ $=\left(\frac{\sqrt{2}}{2}\right)^2-\left(\frac{\sqrt{2}}{2}\right)^2$ $=\frac{2}{4}-\frac{2}{4}$ $h'(\frac{\pi}{4})=0$ SoN: DNE

9. Find the equation of the line tangent to the graph of $g(x)=x^2\cos x$ when $x=\frac{\pi}{2}$.

PoT $\left(\frac{\pi}{2},0\right)$

$g(\frac{\pi}{2})=\left(\frac{\pi}{2}\right)^2\cos\left(\frac{\pi}{2}\right)$

$=\frac{\pi^2}{4}\cdot 0$

$g(\frac{\pi}{2})=0$

SoT: $m=\frac{-\pi^2}{4}$

$g'(x)=2x\cdot\cos x+x^2(-\sin x)$

$g'(x)=2x\cos x-x^2\sin x$

$g'(\frac{\pi}{2})=2(\frac{\pi}{2})\cos(\frac{\pi}{2})-(\frac{\pi}{2})^2\sin(\frac{\pi}{2})$

$=\pi\cdot 0-\frac{\pi^2}{4}(1)$

$g'(\frac{\pi}{2})=\frac{-\pi^2}{4}$

Tangent Line

$y-0=\frac{-\pi^2}{4}\left(x-\frac{\pi}{2}\right)$

Use the table below to complete exercises 10 – 13.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
–2	**1**	**–1**	**2**	**4**
–1	**3**	**–2**	**1**	**1**
0	**–1**	**2**	**–2**	**–3**

10. If $H(x)=2f(x)\cdot g(x)$, what is the value of $\lim\limits_{x\to -2}\frac{H(x)-H(-2)}{x+2}$? $\Rightarrow H'(-2)$ $H'(x)=2f'(x)\cdot g(x)+2f(x)\cdot g'(x)$ $H'(-2)=2f'(-2)g(-2)+2f(-2)\cdot g'(-2)$ $=2(-1)\cdot 2+2(1)\cdot 4$ $=-4+8$ $H'(-2)=4$	11. If $J(x)=g(x)\cdot\sin x$, what is the value of $J'(0)$? $J'(x)=g'(x)\cdot\sin x+g(x)\cdot\cos x$ $J'(0)=g'(0)\cdot\sin(0)+g(0)\cos(0)$ $=(-3)\cdot(0)+(-2)(1)$ $=0-2$ $J'(0)=-2$

12. If $K(x) = (4x - f(x))(2g(x) - 2)$, what is the slope of the normal line when $x = -2$?

SOT

$$\dot{K}(x) = [4 - \dot{f}(x)][2g(x) - 2] + [4x - f(x)][2\dot{g}(x)]$$

$$\dot{K}(-2) = [4 - \dot{f}(-2)][2g(-2) - 2] + [4(-2) - f(-2)][2\dot{g}(-2)]$$

$$= [4 - (-1)][2(2) - 2] + [-8 - (1)][2(4)]$$

$$= [5][4 - 2] + [-9][8]$$

$$= 5 \cdot 2 - 72$$

$$= 10 - 72$$

$$\dot{K}(-2) = -62$$

$$\text{SONL} = \frac{1}{62}$$

Find tangent line

13. If $p(x) = f(x) \cdot \cos x$, what is the equation of the tangent line approximation of the value of $p(0.1)$ using the equation of the tangent line drawn to $p(x)$ at $x = 0$?

PoT: $(0,-1)$

$p(0) = f(0) \cdot \cos(0)$

$= (-1) \cdot 1$

$p(0) = -1$

SoT: $m = 2$

$p'(x) = f'(x)\cos x - f(x)\sin x$

$p'(0) = f'(0)\cos(0) - f(0)\sin(0)$

$= (2)\cdot(1) - (-1)(0)$

$= 2 + 0$

$p'(0) = 2$

Tangent line

$y + 1 = 2(x - 0)$

Estimate $p(0.1)$

$y + 1 = 2(0.1)$

$y + 1 = 0.2$

$y = -0.8$

$\therefore p(0.1) \approx -0.8$