Name________________________________Date__________________Class__________

Homework 3.2

1. Find the equation of the line tangent to the graph of $g(x) = \dfrac{2x^2 - 3x}{3x+1}$ when $x = -1$.

PoT $(-1, -5/2)$

SoT $m = -\frac{1}{4}$

$g(-1) = \dfrac{2(-1)^2 - 3(-1)}{3(-1)+1} = \dfrac{2(1)+3}{-3+1} = \dfrac{2+3}{-2} = 5/-2$

$g'(x) = \dfrac{(4x-3)\cdot(3x+1) - (2x^2-3x)(3)}{(3x+1)^2}$

$g'(-1) = \dfrac{12x^2 - 9x + 4x - 3 - 6x^2 + 9x}{(3x+1)^2} = \dfrac{6x^2 + 4x - 3}{(3x+1)^2} = \dfrac{6(-1)^2 + 4(-1) - 3}{[3(-1)+1]^2}$

$g'(-1) = \dfrac{6(1) - 4 - 3}{[-3+1]^2} = \dfrac{6-7}{[-2]^2} = \dfrac{-1}{4}$

Tangent line

$y + 5/2 = -\frac{1}{4}(x+1)$

2. Find the equation of the line tangent to the graph of $f(\theta) = \tan\theta \sin\theta$ when $\theta = \frac{\pi}{4}$.

PoT $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$

SoT : $m = \frac{3\sqrt{2}}{2}$

$f\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) \cdot \sin\left(\frac{\pi}{4}\right) = 1 \cdot \frac{\sqrt{2}}{2}$

$f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$f(\theta) = \sec^2\theta \cdot \sin\theta + \tan\theta \cos\theta$

$f\left(\frac{\pi}{4}\right) = \sec^2\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{4}\right)$

$= \left(\frac{2}{\sqrt{2}}\right)^2 \cdot \left(\frac{\sqrt{2}}{2}\right) + (1)\cdot\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{4}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$= \frac{4\sqrt{2}}{4} \cdot \frac{2\sqrt{2}}{4}$

$= \frac{6\sqrt{2}}{4}$

$f\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2}$

Tangent line

$y - \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

3. Find the slope of the normal line drawn to the graph of $f(\theta) = \dfrac{3\theta}{\cos\theta}$ when $\theta = \pi$.

SoT

$f'(\theta) = \dfrac{3\cdot\cos\theta + 3\theta\cdot\sin\theta}{\cos^2\theta}$

$f'(\pi) = \dfrac{3\cos(\pi) + 3\pi\cdot\sin(\pi)}{\cos^2(\pi)}$

$= \dfrac{3(-1) + 3\pi\cdot 0}{(1)^2}$

$f'(\pi) = \dfrac{-3}{1}$

SoN: $\frac{1}{3}$

Find the derivative of each of the following functions.

4. $h(x) = \dfrac{x}{x^2+1}$

$$h' = \frac{1\cdot(x^2+1) - x(2x)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$h' = \frac{1-x^2}{(x^2+1)^2}$$

5. $h(x) = \dfrac{x}{\sqrt{x}+1} = \dfrac{x}{x^{1/2}+1}$

$$\frac{d}{dx}h = \frac{1\cdot(\sqrt{x}+1) - x\cdot\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{(\sqrt{x}+1)^2}$$

$$= \frac{\sqrt{x}+1-\frac{1}{2}\sqrt{x}}{(\sqrt{x}+1)^2}$$

$$= \frac{\frac{1}{2}\sqrt{x}+1}{(\sqrt{x}+1)^2}\cdot\frac{2}{2}$$

$$\frac{dh}{dx} = \frac{\sqrt{x}+2}{2(\sqrt{x}+1)^2}$$

6. $g(\theta) = \dfrac{\cos\theta}{\theta^3}$

$$g'(\theta) = \frac{-\sin\theta\cdot\theta^3 - \cos\theta\cdot 3\theta^2}{(\theta^3)^2}$$

$$= \frac{-\theta^3\sin\theta - 3\theta^2\cos\theta}{\theta^6}$$

$$= \frac{\theta^2(-\theta\sin\theta - 3\cos\theta)}{\theta^6}$$

$$g'(\theta) = \frac{-\theta\sin\theta - 3\cos\theta}{\theta^4}$$

7. $f(\theta) = \dfrac{3(1-\sin\theta)}{2\cos\theta} = \dfrac{3-3\sin\theta}{2\cos\theta}$

$$f'(\theta) = \frac{-3\cos\theta\cdot(2\cos\theta) - (3-3\sin\theta)(-2\sin\theta)}{[2\cos\theta]^2}$$

$$= \frac{-6\cos^2\theta - (-6\sin\theta + 6\sin^2\theta)}{4\cos^2\theta}$$

$$= \frac{-6\cos^2\theta + 6\sin\theta - 6\sin^2\theta}{4\cos^2\theta}$$

$$= \frac{-6(\cos^2\theta + \sin^2\theta - \sin\theta)}{4\cos^2\theta}$$

$$= \frac{-3(1-\sin\theta)}{2\cos^2\theta}$$

$$= \frac{-3(1-\sin\theta)}{2(1-\sin^2\theta)}$$

$$= \frac{-3(1-\sin\theta)}{2(1-\sin\theta)(1+\sin\theta)}$$

$$f'(\theta) = \frac{-3}{2(1+\sin\theta)}$$

Use the table below to complete exercises 8 – 10.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
–2	1	–1	2	4
–1	3	–2	1	1
0	–1	2	–2	–3

8. If $H(x) = \dfrac{2f(x)}{g(x)}$, what is the equation of the tangent line when $x = -1$?

PoT: $(-1, 6)$

$$H(-1) = \frac{2f(-1)}{g(-1)} = \frac{2 \cdot 3}{1} = 6$$

SoT: $m = -10$

$$H' = \frac{2f'(x)\,g(x) - 2f(x) \cdot g'(x)}{[g(x)]^2}$$

$$H'(-1) = \frac{2f'(-1)\,g(-1) - 2f(-1)\,g'(-1)}{g^2(-1)}$$

$$= \frac{2(-2)(1) - 2(3)(1)}{(1)^2}$$

$$= \frac{-4 - 6}{1}$$

$$H'(-1) = -10$$

Tangent line

$$\boxed{y - 6 = -10(x+1)}$$

9. If $J(x) = \dfrac{3x + \cos x}{f(x)}$, what is the value of $J'(0)$?

$$J'(x) = \frac{(3 - \sin x) f(x) - (3x + \cos x) f'(x)}{[f(x)]^2}$$

$$J'(0) = \frac{[3 - \sin(0)] \cdot f(0) - [3(0) + \cos(0)] f'(0)}{[f(0)]^2}$$

$$= \frac{[3 - 0] \cdot (-1) - [0 + 1](2)}{(-1)^2}$$

$$= \frac{3(-1) - 2}{1}$$

$$= -3 - 2$$

$$J'(0) = -5$$

10. If $K(x) = \dfrac{4x + f(x)}{3 - g(x)}$, what is the slope of the normal line when $x = -2$?

SoT: $m = -25$

$$K' = \frac{[4 + f'(x)][3 - g(x)] - [4x + f(x)] \cdot [-g'(x)]}{[3 - g(x)]^2}$$

$$K'(-2) = \frac{[4 + f'(-2)][3 - g(-2)] + [4(-2) + f(-2)][g'(-2)]}{[3 - g(-2)]^2}$$

$$= \frac{[4 + (-1)][3 - (2)] + [-8 + (1)][(4)]}{[3 - (2)]^2}$$

$$= \frac{[3][1] + [-7][4]}{[1]^2}$$

$$= \frac{3 - 28}{1}$$

$$K'(-2) = -25$$

SoN: $\frac{1}{25}$