

**Homework 3.3**

In exercises 1 – 6, find the derivative of each of the following functions.

$$1. f(x) = \left( \frac{x+5}{x^2+2} \right)^3$$

*Chain Rule*

$$\begin{aligned} \frac{df}{dx} &= 3 \left( \frac{x+5}{x^2+2} \right)^2 \cdot \frac{(1)(x^2+2) - (x+5)(2x)}{(x^2+2)^2} \\ &= \frac{3(x+5)^2}{(x^2+2)^2} \cdot \frac{x^2+2 - 2x^2 - 10x}{(x^2+2)^2} \\ &= \frac{3(x+5)^2(-x^2 - 10x + 2)}{(x^2+2)^4} \\ &= -\frac{3(x+5)^2(x^2 + 10x - 2)}{(x^2+2)^4} \end{aligned}$$

$$2. f(x) = \sqrt{\frac{2x+3}{x-2}} = \left( \frac{2x+3}{x-2} \right)^{\frac{1}{2}}$$

*Chain Rule*

$$\begin{aligned} f' &= \frac{1}{2} \left( \frac{2x+3}{x-2} \right)^{-\frac{1}{2}} \cdot \frac{2(x-2) - (2x+3)(1)}{(x-2)^2} \\ &= \frac{1}{2} \left( \frac{x-2}{2x+3} \right)^{\frac{1}{2}} \cdot \frac{2x-4 - 2x-3}{(x-2)^2} \\ &= \frac{(x-2)^{\frac{1}{2}}}{2(2x+3)^{\frac{1}{2}}} \cdot \frac{-7}{(x-2)^2} \\ &= \frac{-7}{2(2x+3)^{\frac{1}{2}}(x-2)^{\frac{3}{2}}} \\ f' &= \frac{-7}{2\sqrt{2x+3}\sqrt{(x-2)^3}} \end{aligned}$$

$$3. h(x) = \sqrt{x^2 - 3x + 1} = (x^2 - 3x + 1)^{\frac{1}{2}}$$

*Chain*

$$\begin{aligned} \frac{dh(x)}{dx} &= \frac{1}{2} (x^2 - 3x + 1)^{-\frac{1}{2}} \cdot (2x - 3) \\ &= \frac{2x - 3}{2\sqrt{x^2 - 3x + 1}} \end{aligned}$$

$$4. g(x) = \sqrt[3]{9x^2 + 4} = (9x^2 + 4)^{\frac{1}{3}}$$

*Chain*

$$\begin{aligned} g'(x) &= \frac{1}{3} (9x^2 + 4)^{-\frac{2}{3}} \cdot (18x) \\ g'(x) &= \frac{6x}{3\sqrt[3]{(9x^2 + 4)^2}} \end{aligned}$$

5.  $f(x) = 3x^2 \cos(2x)$

$$f'(x) = \underbrace{(6x \cdot \cos(2x) + 3x^2(-\sin(2x)) \cdot 2)}_{\text{Product Rule}}$$

$$f'(x) = 6x \cos(2x) - 6x^2 \sin(2x)$$

6.  $p(x) = \frac{\cot x}{\sin x}$

Quotient Rule

$$\dot{p}(x) = \frac{(-\csc^2 x) \cdot \sin x - \cot x (\cos x)}{(\sin x)^2}$$

$$= \frac{-\frac{1}{\sin^2 x} \cdot \sin x - \frac{\cos x}{\sin x} \cdot \cos x}{\sin^2 x}$$

$$= \frac{-\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}}{\sin^2 x}$$

$$= \frac{-1 - \cos^2 x}{\sin^3 x}$$

$$= \frac{-1 - (1 - \sin^2 x)}{\sin^3 x}$$

$$= \frac{-1 - 1 + \sin^2 x}{\sin^3 x}$$

$$\dot{p}(x) = \frac{-2 + \sin^2 x}{\sin^3 x}$$

$\left. \begin{array}{l} \text{many correct versions} \\ \text{of this answer} \end{array} \right\}$

For exercises 7 and 8, find the value of the derivative of the function at the given point.

7.  $g(\theta) = \frac{1}{4} \sin^2(2\theta)$  when  $\theta = \pi$

S o T  $m=0$

$$\frac{dg}{d\theta} = \frac{1}{2} \sin(2\theta) \cdot \cos(2\theta) \cdot 2$$

$$= \sin(2\theta) \cdot \cos(2\theta)$$

$$\left. \frac{dg}{d\theta} \right|_{\theta=\pi} = \sin(2\pi) \cdot \cos(2\pi)$$

$$= 0 \cdot 1$$

$$\left. \frac{dg}{d\theta} \right|_{\theta=\pi} = 0$$

8.  $f(\theta) = \sin 2\theta \cos 2\theta$  when  $\theta = \frac{\pi}{4}$

S o T

Product Rule

$$f'(\theta) = \cos(2\theta) \cdot 2 \cdot \cos(2\theta) + \sin(2\theta)(-\sin(2\theta)) \cdot 2$$

$$f'(\theta) = 2 \cos^2(2\theta) - 2 \sin^2(2\theta)$$

$$f'\left(\frac{\pi}{4}\right) = 2 \cos^2\left(2 \cdot \frac{\pi}{4}\right) - 2 \sin^2\left(2 \cdot \frac{\pi}{4}\right)$$

$$= 2 \cos^2\left(\frac{\pi}{2}\right) - 2 \sin^2\left(\frac{\pi}{2}\right)$$

$$= 2(0)^2 - 2(1)^2$$

$$= 0 - 2(1)$$

$$f'\left(\frac{\pi}{4}\right) = -2$$

- ① ②  
9. Find the following limit. Explain the reasoning that you used to arrive at your answer.

$$\lim_{h \rightarrow 0} \frac{\cos 3(x+h) - \cos 3x}{h}$$

② The definition of derivative is  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  so  $f(x) = \cos 3x$ .

①  $f'(x) = -\sin(3x) \cdot 3$

$f'(x) = -3 \sin(3x)$

10. If  $f(x) = \tan 4x$ , then  $f'(x) = \underline{4 \sec^2(4x)}$ .

11. If  $f(\theta) = \sec 5\theta$ , then  $f'(\theta) = \underline{5 \cdot \tan(5\theta) \sec(5\theta)}$ .

12. If  $f(\theta) = \csc 2\theta$ , then  $f'(\theta) = \underline{-2 \cot(2\theta) \csc(2\theta)}$ .

Use the table below to complete exercises 13 – 14.

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	1	-1	2	4
-1	3	-2	1	1
0	-1	2	-2	-3

13. If  $H(x) = \sqrt{f(x) \cdot g(x)}$ , is the graph of  $H(x)$  increasing or decreasing when  $x = -1$ ? Give a reason for your answer.

$$H(x) = [f(x) \cdot g(x)]^{\frac{1}{2}}$$

CHAIN  
PRODUCT

$$\frac{dH}{dx} = \frac{1}{2}[f \cdot g]^{-\frac{1}{2}} \cdot (f'g + fg')$$

$$\frac{dH}{dx} = \frac{f'(x)g(x) + f(x)g'(x)}{2\sqrt{f(x)g(x)}}$$

$$\left. \frac{dH}{dx} \right|_{x=-1} = \frac{f'(-1)g(-1) + f(-1)g'(-1)}{2\sqrt{f(-1)g(-1)}}$$

$$= \frac{(-2) \cdot (1) + (3)(1)}{2\sqrt{(3)(1)}}$$

$$= \frac{-2 + 3}{2\sqrt{3}}$$

$$\left. \frac{dH}{dx} \right|_{x=-1} = \frac{1}{2\sqrt{3}}$$

$H(x)$  is increasing when  $x = -1$   
because  $H'(x) > 0$  at  $x = -1$

14. If  $P(x) = (2f(x) + g(x))^{\frac{2}{3}}$ , what is the value of  $P'(0)$ ?

$$P'(x) = \frac{2}{3} [2f(x) + g(x)]^{\frac{1}{3}} \cdot [2f'(x) + g'(x)]$$

$$P'(x) = \frac{2[2f'(x) + g'(x)]}{3\sqrt[3]{2f(x) + g(x)}}$$

$$P'(x) = \frac{4[f'(x) + 2g'(x)]}{3\sqrt[3]{2f(x) + g(x)}}$$

$$P'(0) = \frac{4f'(0) + 2g'(0)}{3\sqrt[3]{2f(0) + g(0)}}$$

$$= \frac{4(2) + 2(-3)}{3\sqrt[3]{2(-1) + (-1)}}$$

$$= \frac{8 - 6}{3\sqrt[3]{-2 - 2}}$$

$$= \frac{2}{3\sqrt[3]{-4}}$$

$$P'(0) = \frac{-2}{3\sqrt[3]{4}}$$

15. Find the equation of the normal line to the graph of  $h(x) = \tan(3x)$  when  $x = \frac{\pi}{12}$ .

PoT:  $(\frac{\pi}{12}, 1)$

$$h(\frac{\pi}{12}) = \tan(3 \cdot \frac{\pi}{12})$$

$$= \tan(\frac{\pi}{4})$$

$$h(\frac{\pi}{12}) = 1$$

SOT:  $m = 6$

$$h'(x) = 3 \cdot \sec^2(3x)$$

$$h'(\frac{\pi}{12}) = 3 \cdot \sec^2(3 \cdot \frac{\pi}{12})$$

$$= 3 \cdot \sec^2(\frac{\pi}{4})$$

$$= 3 \cdot (\sqrt{2})^2$$

$$= 3 \cdot 2$$

$$h'(\frac{\pi}{12}) = 6$$

SON:  $m = -\frac{1}{6}$

$$y - 1 = -\frac{1}{6}(x - \frac{\pi}{12})$$