

Homework 3.6

For questions 1 – 4, refer to the table of values to the right.

f^{-1}	g^{-1}
(-1, -3)	(1, -3)
(2, 1)	(0, 1)
(-3, 2)	(-2, 2)

x	f	f'	g	g'
-3	-1	2	1	$-\frac{1}{2}$
1	2	3	0	-2
2	-3	$\frac{2}{5}$	-2	-6

1. Find the value of $[f^{-1}(2)]'$.

$$[f^{-1}(x)]' = \frac{1}{f'[f^{-1}(x)]}$$

$$\begin{aligned} [f^{-1}(2)]' &= \frac{1}{f'[f^{-1}(2)]} \\ &= \frac{1}{f'(1)} \\ &= \frac{1}{3} \end{aligned}$$

2. Find the value of $[g^{-1}(1)]'$.

$$[g^{-1}(x)]' = \frac{1}{g'[g^{-1}(x)]}$$

$$\begin{aligned} [g^{-1}(1)]' &= \frac{1}{g'[g^{-1}(1)]} \\ &= \frac{1}{g'(-3)} \\ &= \frac{1}{-\frac{1}{2}} \\ &= -2 \end{aligned}$$

3. Find the equation of the line tangent to the graph of g^{-1} when $x = -2$.

PoT (-2, 2)	SOT : $m = -\frac{1}{6}$	Tangent line
Table	$[g^{-1}(x)]' = \frac{1}{g'[g^{-1}(x)]}$ $[g^{-1}(-2)]' = \frac{1}{g'[g^{-1}(-2)]}$ $= \frac{1}{g'(-3)}$ $= -\frac{1}{6}$	$y - 2 = -\frac{1}{6}(x + 2)$

4. If $h(x) = g(f(x))$, what is the equation of the line normal to the graph of h when $x = 2$?

PoN (2, 1)	SON: $m = 5$	Tangent line
$\begin{aligned} h(2) &= g(f(2)) \\ &= g(1) \\ h(2) &= 1 \end{aligned}$	$\begin{aligned} h'(x) &= g'(f(x)) \cdot f'(x) \\ h'(2) &= g'(f(2)) \cdot f'(2) \\ &= g'(1) \cdot \frac{2}{3} \\ &= -\frac{1}{2} \cdot \frac{2}{3} \\ h'(2) &= -\frac{1}{3} \end{aligned}$	$y - 1 = 5(x - 2)$