NON CALCULATOR PERMITTED

1. If $y = x\sin x$, then $\boxed{\frac{dy}{dx} =}$ $\underbrace{1\cdot\sin x + x\cos x}_{\text{Product Rule}}$

(A) $\sin x + \cos x$

(B) $\sin x + x\cos x$

(C) $\sin x - x\cos x$

(D) $x(\sin x + \cos x)$

(E) $x(\sin x - \cos x)$

2. If $f(x) = 7x - 3 + \ln x$, then $\boxed{f'(1) =}$ $7+\frac{1}{1} = 8$

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

$f'(x) = 7 - 0 + \frac{1}{x} = 7+\frac{1}{x}$

3. If $y = \left(x^3 - \cos x\right)^5$, then $\boxed{y' =}$ $\underbrace{5(x^3-\cos x)^4\cdot(3x^2+\sin x)}_{\text{Chain}}$

(A) $5\left(x^3 - \cos x\right)^4$

(B) $5\left(3x^2 + \sin x\right)^4$

(C) $5\left(3x^2 + \sin x\right)$

(D) $5\left(3x^2 + \sin x\right)^4 \cdot (6x + \cos x)$

(E) $5\left(x^3 - \cos x\right)^4 \cdot \left(3x^2 + \sin x\right)$

4. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the $\boxed{\text{derivative of } f(g(x)) \text{ at } x = 3 \text{ is}}$

(A) $\frac{7}{\sqrt{5}}$ (B) $\frac{14}{\sqrt{5}}$ (C) $\frac{18}{\sqrt{5}}$ (D) $\frac{15}{\sqrt{21}}$ (E) $\frac{30}{\sqrt{21}}$

$f(g(x)) = \sqrt{(3x-2)^2-4}$

$(f(g(x)))' = \underbrace{\cancel{\tfrac{1}{2}}\left((3x-2)^2-4\right)^{-\frac{1}{2}}\cdot\left(\underbrace{\cancel{2}(3x-2)(3)}_{\text{Chain}} - 0\right)}_{\text{Chain}} = \frac{3(3x-2)}{\sqrt{(3x-2)^2-4}}$

$[f(g(3))]' = \frac{3(3\cdot 3-2)}{\sqrt{(3\cdot 3-2)^2-4}} = \frac{3\cdot 7}{\sqrt{7^2-4}} = \frac{21}{\sqrt{45}} = \frac{3\cdot 7}{3\cdot\sqrt{5}} = \frac{7}{\sqrt{5}}$

5. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\frac{1}{2}, \frac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4, \frac{2}{3}\right)$

(E) There are no such points.

PoT

So T: $m = \frac{1}{2}$

$f' = \frac{1 \cdot (x+2) - x(1)}{(x+2)^2}$

$f' = \frac{x+2-x}{(x+2)^2}$

$f' = \frac{2}{(x+2)^2}$

$\frac{1}{2} = \frac{2}{(x+2)^2}$

$x^2 + 4x + 4 = 4$

$x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0$, $x + 4 = 0$, $x = -4$

$x = 0, -4$

6. Let $f(x) = (2x+1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

(A) $-\frac{2}{27}$ (B) $\frac{1}{54}$ (C) $\frac{1}{27}$ (D) $\frac{1}{6}$ (E) 6

f^{-1} $(1,0)$

$g'(1) = [f^{-1}(1)]' = \frac{1}{f'(f^{-1}(1))}$

$= \frac{1}{f'(0)}$

$= \frac{1}{6}$

$f' = 3(2x+1)^2(2)$

$f'(0) = 6(2(0)+1)^2$

$= 6(1)^2$

$f'(0) = 6$

8. The $\lim\limits_{h \to 0} \frac{\ln[\sin(x+h)] - \ln(\sin x)}{h}$ is…

$f(x) = \ln(\sin x) \therefore f' = \frac{\cos x}{\sin x}$

A. $\sin x$ B. x C. $\frac{1}{x}$ D. $\cot x$

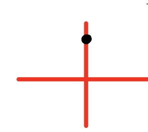
9. The $\lim\limits_{x \to \frac{\pi}{2}} \frac{\sin(x) - \sin\left(\frac{\pi}{2}\right)}{x - \frac{\pi}{2}}$ has a value of …

Alt. Def of f' $\therefore$ $f(x) = \sin x$ Find $f'\left(\frac{\pi}{2}\right)$

$f'(x) = \cos x$

$f'\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2}$

A. 0 B. 1 C. $\frac{\sqrt{2}}{2}$ D. -1



10. $\boxed{\text{The equation of the normal line}}$ to the graph of $y = e^{2x}$ when $\frac{dy}{dx} = 2$ is…

A. $y = -\frac{1}{2}x + 1$

B. ~~$y = 2\left(x - \frac{\ln 2}{2}\right) + 2$~~ SOT

C. ~~$y = 2x + 1$~~ SOT

D. $y = -\frac{1}{2}\left(x - \frac{\ln 2}{2}\right) + 2$

SOT: $m = 2$ PoT$(0,1)$

$\frac{dy}{dx} = 2e^{2x}$

$2 = 2e^{2x}$

$1 = e^{2x}$

$e^0 = e^{2x}$

$0 = 2x$

$x = 0$

$y = e^{2(0)}$

$y = e^0$

$y = 1$

Normal line

$y - 1 = -\frac{1}{2}(x - 0)$

$y - 1 = -\frac{1}{2}x$

$y = -\frac{1}{2}x + 1$

11. If $f(x) = 5\cos^2(\pi - x)$, then $\boxed{f'\left(\frac{\pi}{2}\right) \text{ is} \ldots}$

$= 5[\cos(\pi - x)]^2$

A. 0

B. $-\frac{2}{3}$

C. $\frac{2}{3}$

D. $-\frac{5}{6}$

Chain

Chain

$f'(x) = 10[\cos(\pi - x)]^1 \cdot (-\sin(\pi - x))(-1)$

$f'\left(\frac{\pi}{2}\right) = 10 \cdot \cos\left(\pi - \frac{\pi}{2}\right) \cdot \sin\left(\pi - \frac{\pi}{2}\right)$

$= 10 \cdot \cos\left(\frac{\pi}{2}\right) \sin\left(\frac{\pi}{2}\right)$

$= 10 \cdot 0 \cdot 1$

$= 0$

12. $\boxed{\text{For what value(s) of } k}$ does the graph of $g(x) = ke^{2x} + 3x$ have a normal line whose slope is $-\frac{1}{5}$ when $x = 1$?

SOT $= 5$

$x = 1$

A. e

B. $\frac{1}{e^2}$

C. $-\frac{8}{5e^2}$

D. $\frac{2}{e^2}$

$\frac{dg}{dx} = 2ke^{2x} + 3$

$m = 5$

$x = 1$

$5 = 2ke^{2(1)} + 3$

$5 = 2ke^2 + 3$

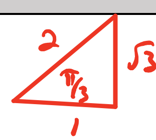
$2 = 2ke^2$

$1 = ke^2$

$\frac{1}{e^2} = k$

13. If $f'(x) = \tan(2x)$, then $\boxed{f'\left(\frac{\pi}{6}\right) =}$ $\tan\left(2 \cdot \frac{\pi}{6}\right)$

$= \tan\left(\frac{\pi}{3}\right)$



A. $2\sqrt{3}$

B. 4

C. $\sqrt{3}$

D. 8

14. If $y=3x\left(3^{-2x}\right)$, then $\boxed{\frac{dy}{dx}=}$ $3\cdot\left(3^{-2x}\right)+3x\cdot\left(3^{-2x}\right)\cdot(-2)\cdot\ln 3$

$= 3\cdot\left(3^{-2x}\right)\left[1-2\ln 3\right)$

$= \dfrac{3(1-2\ln 3)}{3^{2x}}$

A. $-\dfrac{6\ln 3}{3^{2x}}$

B. $\dfrac{3\ln 3}{3^{2x}}$

C. $\dfrac{3(1-2x\ln 3)}{3^{2x}}$

D. $\dfrac{1+x\ln 3}{9^{2x}}$

15. If $f(x)=\log_5(5x+1)^4$, then what is the value of $\boxed{f'(1)?}$

A. $\dfrac{10}{3\ln 5}$

B. $\dfrac{4}{\ln 6}$

C. $\dfrac{2}{3\ln 5}$

D. $\dfrac{4}{\ln 5}$

$f'(x)=\dfrac{4(5x+1)^3(5)}{(5x+1)^4\cdot\ln 5}$

$f'(x)=\dfrac{20}{(5x+1)\ln 5}$

$f'(1)=\dfrac{20}{(5(1)+1)\ln 5}$

$f'(1)=\dfrac{20}{6\ln 5}$

$f'(1)=\dfrac{10}{3\ln 5}$

CALCULATOR PERMITTED

14. The graph of $y=e^{\tan x}-2$ crosses the x-axis at one point in the interval $[0, 1]$. What is the slope of the graph at this point?

(A) 0.606 (B) 2 (C) 2.242 (D) 2.961 (E) 3.747

CALC

$Y_1 = e^{\tan x} - 2$

2nd CALC zero ∴ x-int = 0.606

CALC

2nd CALC $\frac{dy}{dx}$ $x=.606$

∴ $y' = 2.9599$

15. Given that $f(x)=x^2e^x$, what is an approximate value of $f(1.1)$ if you use the equation of the tangent line to the graph of f at $x=1$?

A. 3.534

B. 3.635

C. 7.055

D. 8.155

PoT $(1,e)$

$f(1)=(1)^2e^1$

$f(1)=e$

SoT: $m=3e$

$f'(x)=2x\cdot e^x+x^2e^x$

$f'(1)=2(1)e^1+(1)^2e^1$

$=2e+e$

$f'(1)=3e$

tangent line

$y-e=3e(x-1)$

$y=3ex-3e+e$

$y=3ex-2e$

@ $x=1.1$

$y=3e(1.1)-2e$

$y=3.3e-2e$

$y=1.3e$

$y\approx 3.534$

16. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangents?

A. -0.701 B. -0.567 C. -0.391 D. -0.302

$f' = g'$

$6e^{2x} = 18x^2$

$x \approx -0.391$

CALC

$Y_1 = 6e^{2x}$

$Y_2 = 18x^2$

2nd CALC INTERSECT

17. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$.

~~A. $y = 8x - 5$~~ B. $y = x + 7$ C. $y = x + 0.763$ D. $y = x - 0.122$

SoT: $m=1$	PoT $(0.237, 0.115)$	tangent
$f'(x) = 4x^3 + 4x$ $1 = 4x^3 + 4x$ CALC INTERSECT $x \approx 0.237$	$f(0.237) = (0.237)^4 + 2(0.237)^2$ $= 0.115$	$y - 0.115 = 1(x - 0.237)$ $y - 0.115 = x - 0.237$ $y = x - 0.122$

18. On the interval $-4 < x < 4$, for what value(s) of x will the graphs of $y = \log_4\left(\frac{2x}{2x+3}\right)$ and $y = x^4 + 3xe^x$ have parallel tangent lines?

$y = \log_4(2x) - \log_4(2x+3)$

$y' = \frac{2}{2x \ln 4} - \frac{2}{(2x+3)\ln 4}$

$y' = 4x^3 + \underbrace{(3)e^x + 3x(e^x)}_{\text{product}}$

$y' = 4x^3 + 3e^x + 3xe^x$

A. -0.395 only

B. -1.568 and -0.395

C. -0.480 only

D. -2.234 and -0.480