#1)

x	0	25	30	50
$f(x)$	4	6	8	12

The values of a continuous function f for selected values of x are given in the table above. What is the value of the left Riemann sum approximation to $\int_0^{50} f(x)\,dx$ using the subintervals $[0, 25]$, $[25, 30]$, and $[30, 50]$?

(A) 290 (B) 360 (C) 380 (D) 390 (E) 430

#2) A function $f(t)$ gives the rate of evaporation of water, in liters per hour, from a pond, where t is measured in hours since 12 noon. Which of the following gives the meaning of $\int_4^{10} f(t)\,dt$ in the context described?

(A) The total volume of water, in liters, that evaporated from the pond during the first 10 hours after 12 noon

(B) The total volume of water, in liters, that evaporated from the pond between 4 P.M. and 10 P.M.

(C) The net change in the rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.

(D) The average rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.

(E) The average rate of change in the rate of evaporation, in liters per hour per hour, from the pond between 4 P.M. and 10 P.M.

#3) Let f be the function given by $f(x) = 9^x$. If four subintervals of equal length are used, what is the value of the right Riemann sum approximation for $\int_0^2 f(x)\,dx$?

(A) 20 (B) 40 (C) 60 (D) 80 (E) 120

#4)

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24

Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \le 12$ seconds, Kathleen's velocity k, measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.

(a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t)\,dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t)\,dt$? Explain your reasoning.

(c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \dfrac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

(d) Write an expression for Nathan's acceleration in terms of t.

#5)

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

(a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

(b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

(c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

(d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

Riemann Sum

Name______________________________

#6)

t (weeks)	0	3	6	10	12
$G(t)$ (games per week)	160	450	900	2100	2400

A store tracks the sales of one of its popular board games over a 12-week period. The rate at which games are being sold is modeled by the differentiable function G, where $G(t)$ is measured in games per week and t is measured in weeks for $0 \le t \le 12$. Values of $G(t)$ are given in the table above for selected values of t.

(a) Approximate the value of $G'(8)$ using the data in the table. Show the computations that lead to your answer.

(b) Approximate the value of $\int_0^{12} G(t)\,dt$ using a right Riemann sum with the four subintervals indicated by the table. Explain the meaning of $\int_0^{12} G(t)\,dt$ in the context of this problem.

Riemann Sum

Name________________________________

#7)

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$ hours, as shown in the table above. For $0 < t \le 2.4$, $v(t) > 0$.

(a) Use the data in the table to approximate Ruth's acceleration at time $t = 1.4$ hours. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

AP Calculus	Riemann Sum	Name_______________________________

7.1 – 7.2 Test Prep

MC

#1) A

#2) B

#3) C

Free Response

#4)

(a) $a(4) \approx \frac{10-5}{5-3} = \frac{5}{2}$ ft/sec^2	$2: \begin{cases} 1: \text{estimate} \\ 1: \text{units} \end{cases}$
(b) $\int_0^{12} k(t)\,dt \approx (5)(3) + (10)(2) + (20)(3) + (24)(4) = 191$ feet This approximation is an overestimate since a right Riemann sum is used and the function k is increasing.	$3: \begin{cases} 1: \text{right Riemann sum} \\ 1: \text{approximation with units} \\ 1: \text{overestimate with reason} \end{cases}$
(c) $s(12) = 5 + \int_0^{12} n(t)\,dt$	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(d) $n'(t) = (150)(-1)(t+3)^{-2} - 50e^{-t}(-1)$ $= -\frac{150}{(t+3)^2} + 50e^{-t}$	$2: n'(t)$

#5)

(a) An equation of the tangent line is $y = 0.5(t-30) + 125$. $W(32) \approx 0.5(32-30) + 125 = 126$	1 : answer
(b) $\int_0^{30} W'(t)\,dt \approx (10)(0.6) + (12)(0.7) + (8)(1.0) = 22.4$ $W(0) = W(30) - \int_0^{30} W'(t)\,dt = 125 - 22.4 = 102.6$	$3: \begin{cases} 1: \text{left Riemann sum} \\ 1: \text{approximation} \\ 1: \text{answer} \end{cases}$
(c) W' is differentiable $\Rightarrow$ W' is continuous. $W'(30) = 0.5 < 0.7 < 1.0 = W'(22)$ By the Intermediate Value Theorem, there must be at least one time t, $22 \le t \le 30$, such that $W'(t) = 0.7$.	2 : explanation
(d) $\frac{dA}{dt} = (0.3)\frac{2}{3}W^{-1/3} \cdot \frac{dW}{dt} = \frac{0.2}{\sqrt[3]{W}} \cdot \frac{dW}{dt}$ $\left.\frac{dA}{dt}\right\vert_{t=30} = \frac{0.2}{\sqrt[3]{125}} \cdot 0.5 = 0.02$	$3: \begin{cases} 2: \frac{dA}{dt} \\ 1: \text{answer} \end{cases}$

#6)

(a) $G'(8) \approx \frac{G(10) - G(6)}{10 - 6} = \frac{2100 - 900}{10 - 6} = 300$ games per week per week

1 : approximation

(b) $\int_0^{12} G(t)\,dt \approx 3 \cdot G(3) + 3 \cdot G(6) + 4 \cdot G(10) + 2 \cdot G(12)$
$= 3 \cdot 450 + 3 \cdot 900 + 4 \cdot 2100 + 2 \cdot 2400$
$= 17250$

$\int_0^{12} G(t)\,dt$ represents the total number of games sold over the 12-week period $0 \le t \le 12$.

3 : { 1 : right Riemann sum; 1 : approximation; 1 : explanation }

#7)

(a) $a(1.4) \approx \frac{v(1.6) - v(1.2)}{1.6 - 1.2} = \frac{16.3 - 17.2}{1.6 - 1.2} = -2.25$ miles/hr^2

2 : { 1 : approximation; 1 : units }

(b) $\int_0^{2.4} v(t)\,dt$ is the total distance Ruth traveled, in miles, from time $t = 0$ to time $t = 2.4$ hours.

$\int_0^{2.4} v(t)\,dt \approx (0.8)(11.8) + (0.8)(17.2) + (0.8)(16.8)$
$= 36.64$ miles

3 : { 1 : interpretation; 1 : midpoint Riemann sum; 1 : approximation }