Use the graph to answer 1-3.

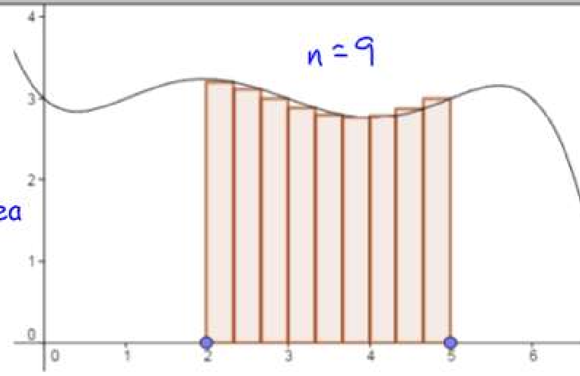
1. Is the rectangular approximation shown to the right a left endpoint, right endpoint, or midpoint approximation?

2. Is the approximation less than or greater than the true value?

slightly less, more area missing under the curve than extra area above the curve

3. What is the width of each rectangle?

$\Delta x = \frac{b-a}{n} = \frac{5-2}{9} = \frac{3}{9} = \frac{1}{3}$

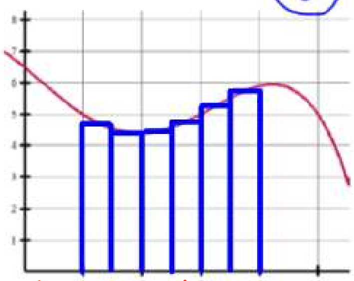


$n=9$

Sketch the following rectangular approximations. Find the width of each subinterval.

4. Midpoint on the interval [1,4] with $n = 6$ subintervals

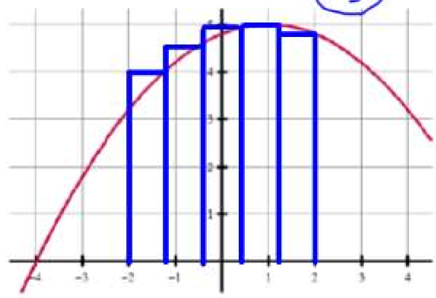
Width of each subinterval = $\frac{1}{2}$



$\Delta x = \frac{b-a}{n} = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}$

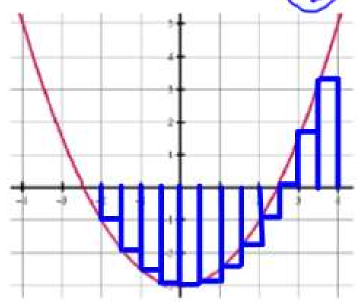
5. Right Endpoint on [-2,2] with $n = 5$ subintervals

Width of each subinterval = $\frac{4}{5}$



6. Left Endpoint on [-2,4] with $n = 12$ subintervals

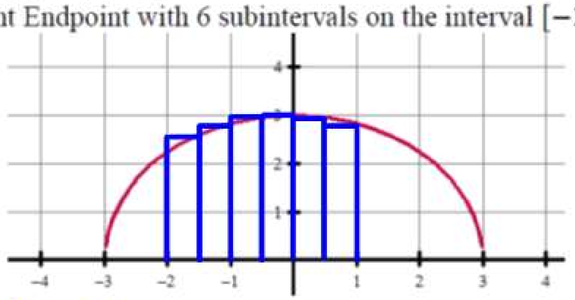
Width of each subinterval = $\frac{1}{2}$



Approximate the area under the curve using the given rectangular approximation. Include a sketch! Justify!

7. $f(x) = \sqrt{9 - x^2}$

Right Endpoint with 6 subintervals on the interval $[-2,1]$



$\Delta x = \frac{b-a}{n} = \frac{1-(-2)}{6} = \frac{3}{6} = \frac{1}{2}$

$A \approx \Delta x(f(-1.5) + f(-1) + f(-0.5) + f(0) + f(0.5) + f(1))$

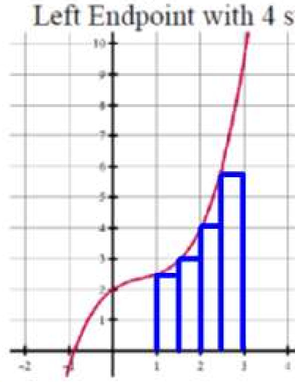
$A \approx \frac{1}{2}(2.598 + 2.828 + 2.958 + 3 + 2.958 + 2.828)$

$A \approx \frac{1}{2}(17.171)$

$A \approx 8.585 \text{ un}^2$

8. $f(x) = \frac{1}{2}x^3 - x^2 + x + 2$

Left Endpoint with 4 subintervals on the interval [1,3]

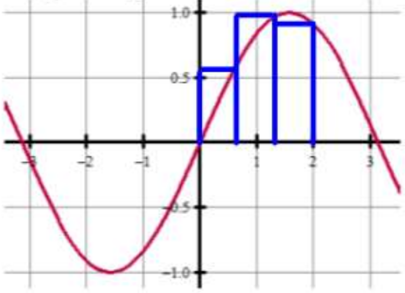


$A \approx 7.75 \text{ un}^2$

Approximate the area under the curve using the given rectangular approximation. Include a sketch! Justify!

9. $f(x) = \sin x$

Right Endpoint with 3 subintervals on the interval $[0,2]$



$\Delta x = \frac{b-a}{n}$

$= \frac{2-0}{3}$

$\Delta x = \frac{2}{3}$

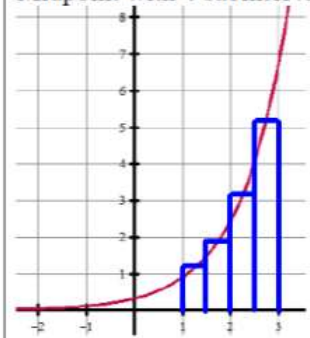
$A \approx \Delta x\left(f\left(\frac{2}{3}\right) + f\left(\frac{4}{3}\right) + f(2)\right)$

$\approx \frac{2}{3}(0.618 + 0.972 + 0.909)$

$\approx \frac{2}{3}(2.499) = 1.666 \text{ un}^2$

10. $f(x) = \frac{e^x}{3}$

Midpoint with 4 subintervals on the interval $[1,3]$



$A \approx 5.729 \text{ un}^2$

Use the information provided to answer the following.

11. Let $y(t)$ represent the rate of change of the population of a town over a 20-year period, where y is a differentiable function of t. The table shows the population change in people per year recorded at selected times.

Time (years)	0	4	10	13	20
$y(t)$ (people per year)	2500	2724	3108	3697	4283

a. Use the data from the table and a right Riemann Sum with four subintervals to approximate the area under the curve.

People $= 4(2724) + 6(3108) + 3(3697) + 7(4283) = 70616$

b. What does your answer from part (a) represent?

the total people that moved to the town over the 20-year period

c. Assuming that $y(t)$ is a continuous increasing function, is your approximation from part (a) greater or less than the true value?

greater than the true value

12. A rectangular pool gets deeper from one end of the pool to the other. The table shows the depth $h(x)$ of the water at 4 foot intervals from one end of the pool to the other.

position, x (feet)	0	4	8	12	16	20	24	28	32
$h(x)$ (feet)	6.5	8	9.5	10	11	11.5	12	13	13.5

a. Use the data from the table to find an approximation for $h'(10)$, and explain the meaning of $h'(10)$ in terms of the population of the town. Show the computations that lead to your answer.

$h'(10) \approx \text{ARC} = \frac{10-9.5}{12-8} = \frac{0.5}{4} = \frac{1}{8} = 0.125$ feet per foot

b. Use a midpoint Riemann Sum with 4 subintervals to approximate the area under the curve.

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13. Particle A moves along a horizontal line with velocity $v(t)$, where $v(t)$ is a positive continuous function of t. The time t is measured in cm/sec. The velocity of the particle at selected times is given in the table.

t (sec)	0	2	5	7	10
$v(t)$ (cm/sec)	1.7	6.8	7.4	15.6	24.9

a. Use the data from the table to approximate the distance traveled by a particle A over the interval $0 \le t \le 10$ seconds by using a left Riemann Sum with four subintervals. Show the computations that lead to your answer.

cm ≈ $2(1.7) + 3(6.8) + 2(7.4) + 3(15.6) = 85.4$ cm

b. Assuming that $v(t)$ is a continuous increasing function, is the approximation greater or less than the true value?

less than the true value

c. Particle B moves along the same horizontal line with position $x(t) = te^{\sin 3t}$. Which particle is traveling faster at time $t = 5$? Explain your answer.

particle B is travelling faster

A $v(5) = 7.4$ cm/sec

B $x'(5) = v(5) \approx -19.918$ cm/sec

speed = |velocity|

$19.918 > 7.4$

$x'(t) = t' \cdot e^{\sin 3t} + t \cdot (e^{\sin 3t})'$

$= 1 \cdot e^{\sin 3t} + t \cdot (\sin 3t)' \cdot e^{\sin 3t}$

$= e^{\sin 3t} + t \cos(3t) \cdot 3 \cdot e^{\sin 3t}$

$x'(t) = e^{\sin 3t} + 3t \cos(3t) e^{\sin 3t}$

$x'(5) \approx -19.92$