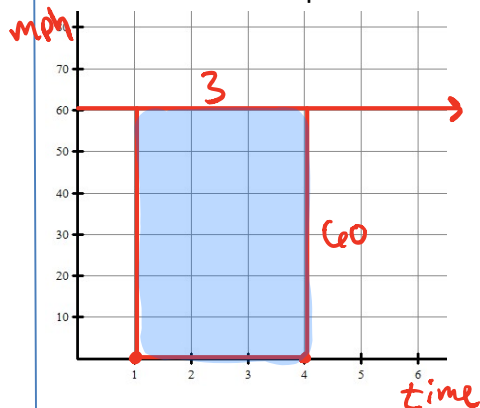# 7.1 Rectangular Approximation

NOTES

CALCULUS

Write your
questions here!
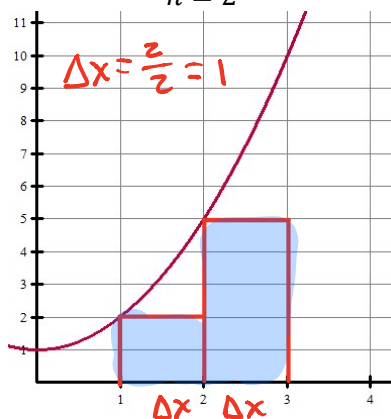
Car travels 60 miles per hour

mph

3

60

time

How far did you travel
from 1 hour to 4 hours? 180 miles

Left Endpoint Rectangle for interval [1,3] with n subintervals
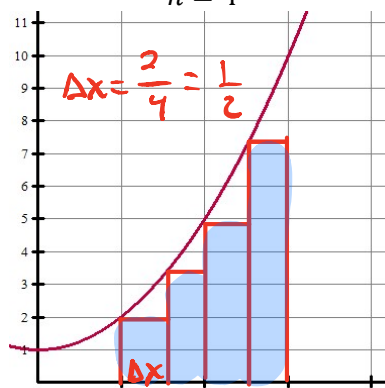
$$\Delta x = \frac{b-a}{n}$$

$$f(x) = x^2 + 1$$

$n = 2$

$$\Delta x = \frac{2}{2} = 1$$

Δx Δx

$$Area = 1 \cdot f(1) + 1 \cdot f(2)$$
$$= 1 \cdot 2 + 1 \cdot 5$$
$$= 7$$

$n = 4$

$$\Delta x = \frac{2}{4} = \frac{1}{2}$$

Δx

$$A \approx \frac{1}{2} \cdot f(1) + \frac{1}{2} f(\tfrac{3}{2}) + \frac{1}{2} f(2) + \frac{1}{2} f(\tfrac{5}{2})$$
$$A \approx \frac{1}{2} \left[f(1) + f(\tfrac{3}{2}) + f(2) + f(\tfrac{5}{2}) \right]$$
$$A \approx \frac{1}{2} \left[2 + \tfrac{13}{4} + 5 + \tfrac{29}{4} \right)$$
$$A \approx \frac{1}{2} \left[\tfrac{35}{2} \right] \approx \frac{35}{4}$$
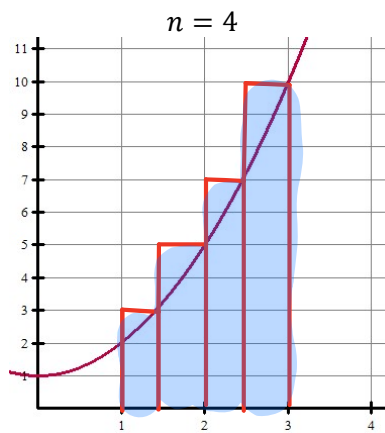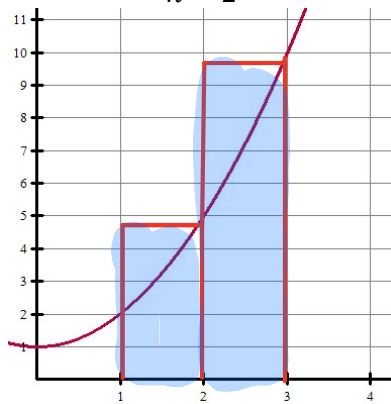
$n = 6$

Δx
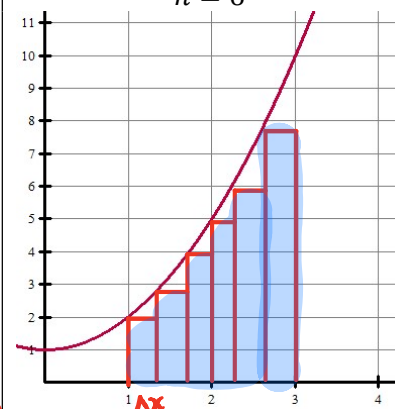
$$\Delta x = \frac{2}{6} = \frac{1}{3}$$

Right Endpoint Rectangle for interval [1,3] with n subintervals

$$f(x) = x^2 + 1$$

$n = 2$
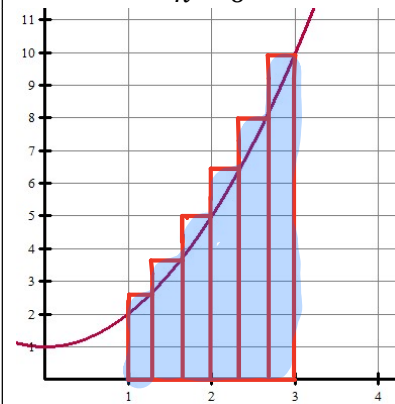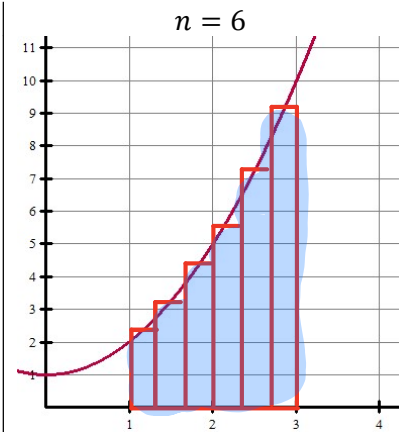

$n = 4$
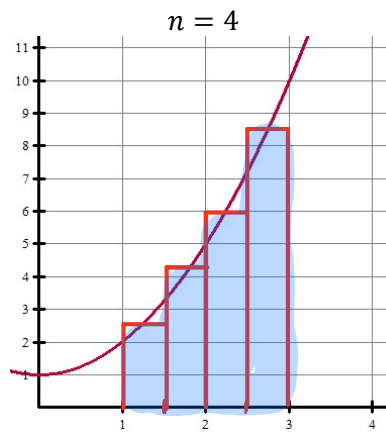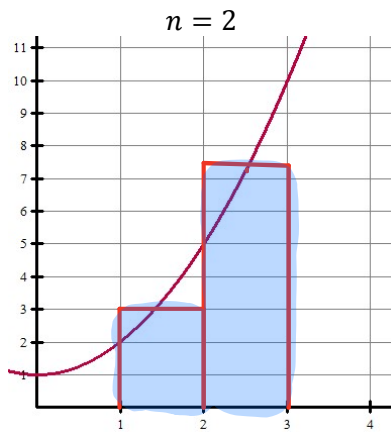

$$A \approx \frac{1}{2} \left(f(1.5) + f(2) + f(2.5) + f(3) \right]$$
$$\approx \frac{1}{2} \left(\tfrac{13}{4} + 5 + \tfrac{29}{4} + 10 \right]$$
$$\approx \frac{1}{2} \left(\tfrac{51}{2} \right)$$
$$\approx \frac{51}{4}$$

$n = 6$

Midpoint Rectangle for interval [1,3] with n subintervals

$$f(x) = x^2 + 1$$

$n = 2$	$n = 4$	$n = 6$

$A \approx \frac{1}{2}\left[f(1.25) + f(1.75) + f(2.25) + f(2.75)\right]$

CALCulator

$A \approx \frac{1}{2}\left[21.25\right]$

$A \approx 10.625$

Sketch the following rectangular approximations

Left Endpoint $n = 5$ [1,3]	Right Endpoint $n = 8$ [1,4]	Midpoint $n = 6$ [2,4]

$\Delta x = \frac{b-a}{n} = \frac{3-1}{5} = \frac{2}{5}$

$\Delta x = \frac{4-1}{8} = \frac{3}{8}$

$\Delta x = \frac{4-2}{6} = \frac{1}{3}$

Reimann Sum

$A = \frac{2}{5}\left[f(1) + f(\frac{7}{5}) + f(\frac{9}{5}) + f(\frac{4}{5})\right]$
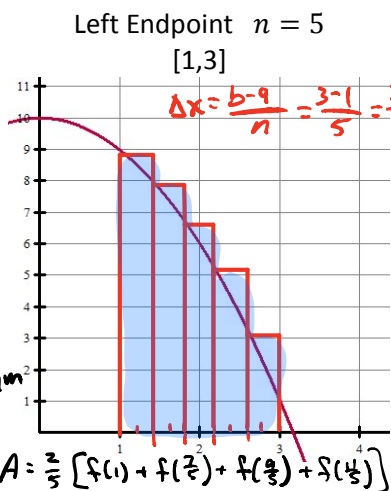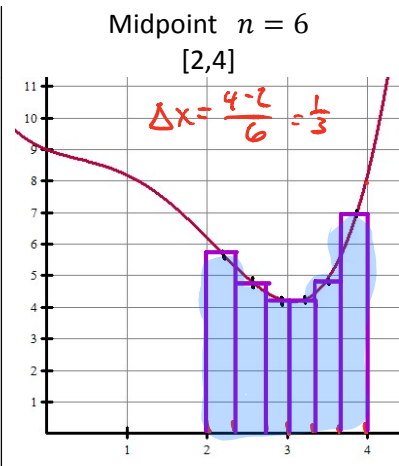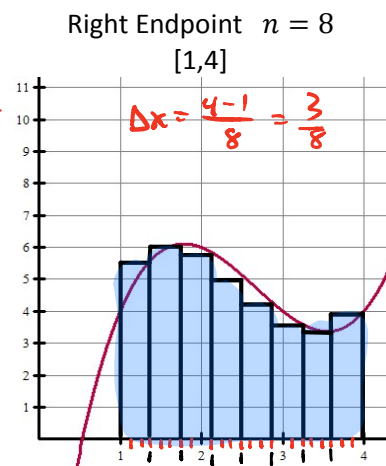
The rate at which water is being pumped into a tank is given by the continuous and increasing function $R(t)$. A table of selected values of $R(t)$, for the time interval $0 < t < 13$ minutes, is given below.

Time (minutes)	0	4	6	10	13
$R(t)$ (gallons/min)	7	13	18	23	27

Use right Riemann Sum with 4 subintervals to approximate the area under the curve.

$A = 4f(4) + 2f(6) + 4f(10) + 3f(13)] \approx 4(13) + 2(18) + 4(23) + 3(27)$

≈ 261

What does this represent?

Total Gallons of water pumped into tank in first 13 minutes.

Is the approximation greater or less than the true value?

Since the graph is increasing, the approximation is greater than the true value.