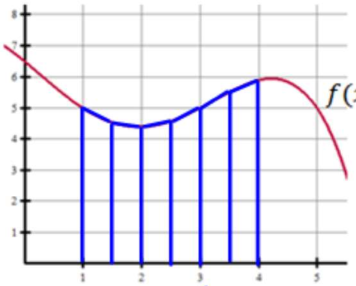
You can use a calculator on 1-13

Sketch the trapezoidal approximations. Find the width of each subinterval. Write a definite integral.

1. $n = 6$ subintervals on[1,4]

$\Delta x = \frac{b-a}{n} =$

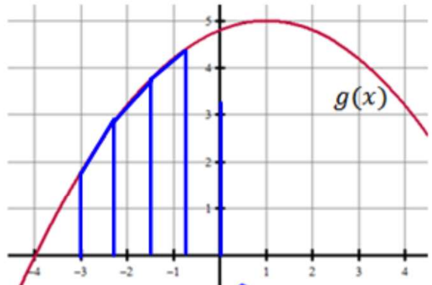
Width of each subinterval = $\frac{4-1}{6} = \frac{1}{2}$



Definite Integral = $\int_1^4 f(x)\,dx$

2. $n = 4$ subintervals on[-3,0]

Width of each subinterval = $\frac{3}{4}$

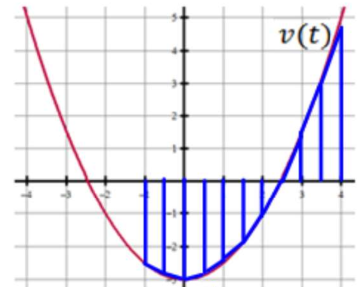


Definite Integral = $\int_{-3}^0 g(x)\,dx$

3. $n = 10$ subintervals on[-1,4]

$\Delta x = \frac{b-a}{n} =$

Width of each subinterval = $\frac{4-(-1)}{10} = \frac{1}{2}$



Definite Integral = $\int_{-1}^4 v(t)\,dt$

Approximate the definite integral use trapezoidal approximation.

4. $\int_{-2}^{4} (9 + x^2)dx$

Trapezoidal approximation with 6 subintervals

79

5. $\int_0^2 \left(\frac{4 - x}{x + 5}\right) dx$

$\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2}$

Trapezoidal approximation with 4 subintervals

$A \approx \frac{1}{2}\left[\frac{f(0)+f(0.5)}{2} + \frac{f(0.5)+f(1)}{2} + \frac{f(1)+f(1.5)}{2} + \frac{f(1.5)+f(2)}{2}\right]$

$f(0) = 0.8$
$f(0.5) = 0.63636$
$f(1) = 0.5$
$f(1.5) = 0.38462$
$f(2) = 0.28571$

$A \approx 1.032 \text{ un}^2$

6. $\int_1^2 (3\sqrt{x + 2})dx$ $\Delta x = \frac{b-a}{n} = \frac{2-1}{3} = \frac{1}{3}$

Trapezoidal approximation with 3 subintervals

5.706

Use the calculator to find the exact value of the definite integral.

7. $\int_1^3 \left(\frac{1}{5}x^3 - 2x^2 + 10\right) dx$

$\approx 6.\overline{6}$

8. $\int_{-2}^{0} (e^x + 4)dx$

≈ 8.8646

9. $\int_{\pi}^{\frac{3\pi}{2}} (\sin^2 x)dx$

≈ 0.785

10. $\int_{-2}^{e} \left(4 - \sqrt{9 - x^2}\right) dx$

≈ 6.525

Use the information provided to answer the following.

11. Let $y(t)$ represent the rate of change of the population of a town over a 20-year period, where y is a differentiable function of t. The table shows the population change in people per year recorded at selected times. The population at $t = 0$ was 25,500 people.

Time (years)	0	4	10	13	20
$y(t)$ (people per year)	2500	2724	3108	3697	4283

a. Use a trapezoidal approximation with four subintervals to approximate $\int_0^{20} y(t)\,dt$

people $\approx 4\left(\frac{2500+2724}{2}\right) + 6\left(\frac{2724+3108}{2}\right) + 3\left(\frac{3108+3697}{2}\right) + 7\left(\frac{3697+4283}{2}\right)$

people $\approx 4(2612) + 6(2916) + 3(3402.5) + 7(3990)$

people $\approx$ 66081.5

b. What is the approximate population after 20 years?

$66081.5 + 25500 =$ 91581.5 people

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12. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by the twice-differentiable and strictly increasing function R of time t. A table of selected values of $R(t)$ for the time interval $0 \le t \le 90$ minutes is shown below. At $t = 0$ the plane had already consumed 84 gallons of fuel.

Time (minutes)	0	30	40	50	70	90
$R(t)$ (gallons per min)	20	30	40	55	65	70

a. Use data from the table to find an approximation for $R'(45)$. Show the computations that led to your answer. Indicate units of measure.

$R'(45) \approx ARC = \frac{55-40}{50-40} = \frac{3}{2}$ gallons/min^2

b. Approximate the value of $\int_0^{90} R(t)dt$ using trapezoidal approximation with five subintervals indicated by the data in the table.

$\int_0^{90} R(t)dt \approx 30\cdot\frac{1}{2}(20+30) + 10\cdot\frac{1}{2}(30+40) + 10\cdot\frac{1}{2}(40+55) + 20\frac{1}{2}(55+65) + 20\cdot\frac{1}{2}(65+70)$

$\approx 15(50) + 5(70) + 5(95) + 10(120) + 10(135)$

≈ 4125

c. Approximately how much fuel has the plane consumed after 90 minutes?

$4125 + 84 = 4209$ gallons of fuel

CALCULATOR ACTIVE

FREE RESPONSE

Your score: ____ out of 6

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table below.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(a) Midpoint Riemann sum is

$$10\cdot[v(5)+v(15)+v(25)+v(35)]$$
$$= 10\cdot[9.2+7.0+2.4+4.3] = 229$$

The integral gives the total distance in miles that the plane flies during the 40 minutes.

$$3: \begin{cases} 1: v(5)+v(15)+v(25)+v(35) \\ 1: \text{answer} \\ 1: \text{meaning with units} \end{cases}$$

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(b) By the Mean Value Theorem, $v'(t) = 0$ somewhere in the interval $(0, 15)$ and somewhere in the interval $(25, 30)$. Therefore the acceleration will equal 0 for at least two values of t.

$$2: \begin{cases} 1: \text{two instances} \\ 1: \text{justification} \end{cases}$$

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.

(c) $f'(23) = -0.407$ or -0.408 miles per minute^2

1 : answer with units