7.2 Trapezoidal Approximation

NOTES

CALCULUS

Write your questions here!

Riemann Sum

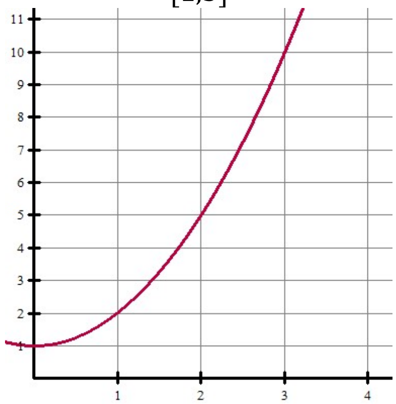
$n = \infty$

$A = h \cdot b$

$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i) \cdot \Delta x = \int f(x)\,dx$

$f(x) = x^2 + 1$

[1,3]

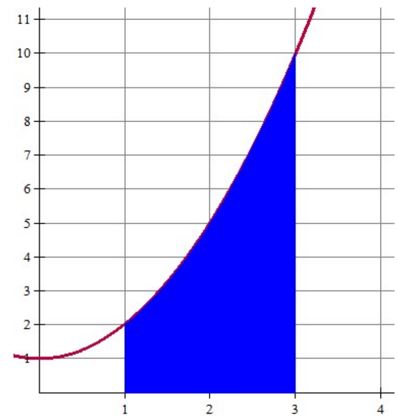


c_1

CALC math fnint

fnint(x^2+1, x, 1, 3)

$f(x) = x^2 + 1$



The Definite Integral

upper limit

integrand

$\int_a^b f(x)\,dx$

variable of integration

Lower limit

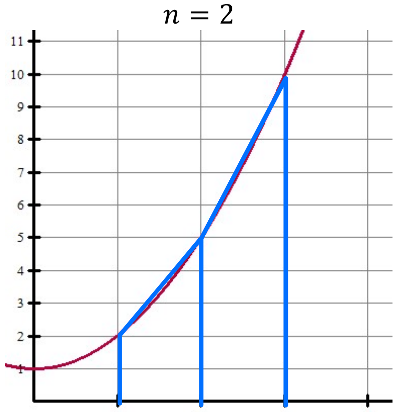
$\text{Area} = \int_1^3 (x^2+1)\,dx = 10.667$

Trapezoidal Approximation for interval [1,3] with *n* subintervals

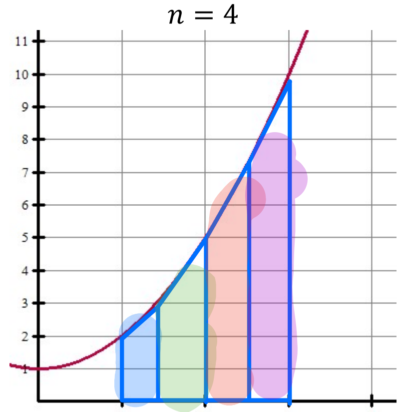
How close is it the true value?

$f(x) = x^2 + 1$

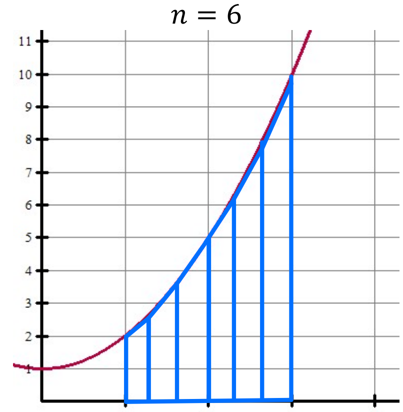
$n = 2$

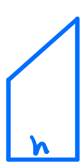


$n = 4$



$n = 6$





b_1 b_2 h

$A_{trap} = h\,\frac{1}{2}(b_1 + b_2)$

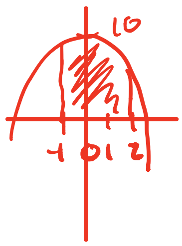
$\text{Area} \approx \frac{1}{2}h(f(1)+f(1.5)) + \frac{1}{2}h(f(1.5)+f(2)) + \frac{1}{2}h(f(2)+f(2.5)) + \frac{1}{2}h(f(2.5)+f(3))$

$\approx \frac{1}{2}(\frac{1}{2})(2+3.25) + \frac{1}{2}(\frac{1}{2})(3.25+5) + \frac{1}{2}(\frac{1}{2})(5+7.25) + \frac{1}{2}(\frac{1}{2})(7.25+10)$

$\approx \frac{1}{4}(5.25) + \frac{1}{4}(8.25) + \frac{1}{4}(12.25) + \frac{1}{4}(17.25)$

$\approx 10.75\ \text{un}^2$

(concave up, overestimate)



$h = \Delta x = \frac{b-a}{n} = \frac{2-(-1)}{3} = \frac{3}{3} = 1$

Given the definite integral $\int_{-1}^{2} (10 - x^2)dx$

(a) Use the Trapezoid Rule with three equal subintervals to approximate its value.

$\text{Area} \approx \frac{1}{2}\Delta x\left(f(-1)+f(0)\right) + \frac{1}{2}\Delta x\left(f(0)+f(1)\right) + \frac{1}{2}\Delta x\left(f(1)+f(2)\right)$

$\approx \frac{1}{2}(1)(9+10) + \frac{1}{2}(1)(10+9) + \frac{1}{2}(1)(9+6)$

$\approx \frac{1}{2}(19) + \frac{1}{2}(19) + \frac{1}{2}(15)$

$\approx \frac{1}{2}(53)$

$\approx 26.5 \text{ un}^2$

(b) Is your answer from part (a) an overestimate or underestimate?

Concave down

(c) Calculate the exact value on your calculator. Compare to your answer in part (a).

Area $= 27$ un^2. Our answer was $\frac{1}{2}$ unit^2 too small

The rate at which customers are being served at Starbrusts is given by the continuous $R(t)$. A table of selected values of $R(t)$, for the time interval $0 < t < 10$ hours, is given below. At $t = 0$ there had been 200 customers served.

Time (hours)	0	2	3	6	10
$R(t)$ (people/hour)	37	44	36	42	48

Use a trapezoidal sum with 4 subintervals to approximate $\int_{0}^{10} R(t)dt$

Total People $= \frac{1}{2}\Delta x\left(f(0)+f(2)\right) + \frac{1}{2}\Delta x\left(f(2)+f(3)\right) + \frac{1}{2}\Delta x\left(f(3)+f(6)\right) + \frac{1}{2}\Delta x\left(f(6)+f(10)\right)$

$= \frac{1}{2}(2)[37+44] + \frac{1}{2}(1)[44+36] + \frac{1}{2}(3)[36+42] + \frac{1}{2}(4)[42+48]$

$= 1[81] + \frac{1}{2}[80] + \frac{3}{2}[78] + 2[90]$

$= 81 + 40 + 117 + 180$

$= 418$ people

Approximately how many customers had been served after 10 hours?

$418 + 200 = 618$ people

SUMMARY:

Now, summarize your notes here!