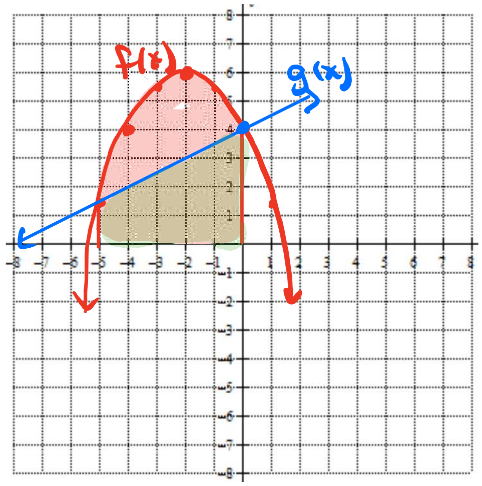
Notes 7.5 – Finding the Area between Two Curves

An Application of Integration

Graph the function $f(x) = -\frac{1}{2}x^2 - 2x + 4$ and find the value of $\int_{-5}^{0} f(x)dx$. Using one color, shade the region for which this value represents the area.



graph: $f'(x) = -x - 2$

$0 = -x - 2$

$x = -2$

$f(-2) = -\frac{1}{2}(-2)^2 - 2(-2) + 4$

$= -2 + 4 + 4$

$f(-2) = 6$

$\int_{-5}^{0}\left(-\frac{1}{2}x^2 - 2x + 4\right)dx = \left[-\frac{1}{6}x^3 - x^2 + 4x\right]\Big|_{-5}^{0}$

$= \left[-\frac{1}{6}(0)^3 - (0)^2 + 4(0)\right] - \left[-\frac{1}{6}(-5)^3 - (-5)^2 + 4(-5)\right]$

$= [0] - \left[\frac{+125}{6} - 25 - 20\right]$

$= \frac{-125}{6} + 45$

≈ 24.167

Graph the function $g(x) = \frac{1}{2}x + 4$ on the same grid and then find the value of $\int_{-5}^{0} g(x)dx$. Using a different color, shade the region for which this value represents the area.

$\int_{-5}^{0}\left(\frac{1}{2}x + 4\right)dx = \left(\frac{1}{4}x^2 + 4x\right)\Big|_{-5}^{0}$

$= \left[\frac{1}{4}(0)^2 + 4(0)\right] - \left[\frac{1}{4}(-5)^2 + 4(-5)\right]$

$= [0] - \left[\frac{25}{4} - 20\right]$

$= -\left[\frac{25}{4} - \frac{80}{4}\right]$

$= +\left[\frac{+55}{4}\right]$

$= 13.75$

What do you suppose you would do to find the area of the region that is located in between the graphs of $f(x)$ and $g(x)$? Find this value.

Area between f(x) and g(x) = 24.167 − 13.75

= 10.417

Now, find the value of the definite integral below if $f(x) = -\frac{1}{2}x^2 - 2x + 4$ and $g(x) = \frac{1}{2}x + 4$. Show your work.

$$\int_{-5}^{0}[f(x) - g(x)]dx$$

$= \int_{-5}^{0}\left[\left(-\frac{1}{2}x^2 - 2x + 4\right) - \left(\frac{1}{2}x + 4\right)\right]dx$

$= \int_{-5}^{0}\left[-\frac{1}{2}x^2 - \frac{5}{2}x\right]dx$

$= \left[-\frac{1}{6}x^3 - \frac{5}{4}x^2\right]\Big|_{-5}^{0}$

$= \left[-\frac{1}{6}(0)^3 - \frac{5}{4}(0)^2\right] - \left[-\frac{1}{6}(-5)^3 - \frac{5}{4}(-5)^2\right]$

$= [0] - \left[\frac{125}{6} - \frac{125}{4}\right]$

$= \frac{-125}{6} + \frac{125}{4}$

$= 10.417$

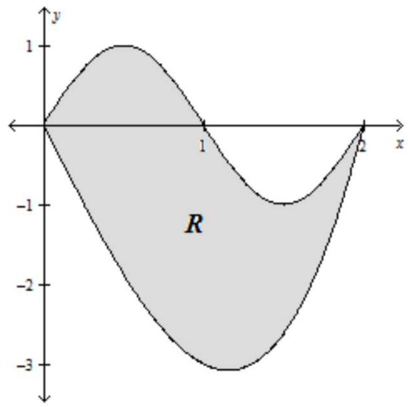
What do you notice about this value?

This brings about the general way that we will find the area between two curves.

Area between $f(x)$ and $g(x)$ $= \int_a^b [f(x) - g(x)]\,dx$

where $x = a$ and $x = b$ are points of intersection of f and g and where $f(x) > g(x)$

Find the area of the shaded region, R, that is bounded by $y = \sin(\pi x)$ and $y = x^3 - 4x$.



$$\boxed{\begin{aligned} u &= \pi x \\ du &= \pi\,dx \\ \frac{du}{\pi} &= dx \end{aligned}}$$

$$\int_0^2 \left[\sin(\pi x) - (x^3 - 4x)\right] dx$$

$$= \int_0^2 \sin(\pi x)\,dx - \int_0^2 (x^3 - 4x)\,dx$$

$$= \int_{\pi(0)}^{2\cdot\pi} \sin(u)\frac{du}{\pi} - \left[\frac{1}{4}x^4 - 2x^2\right]\Big|_0^2$$

$$= \frac{1}{\pi}\int_0^{2\pi} \sin u\,du - \left(\left[\frac{1}{4}(2)^4 - 2(2)^2\right] - [0]\right)$$

$$= -\frac{1}{\pi}\cos u\Big|_0^{2\pi} - \left(\frac{1}{4}(16) - 8\right)$$

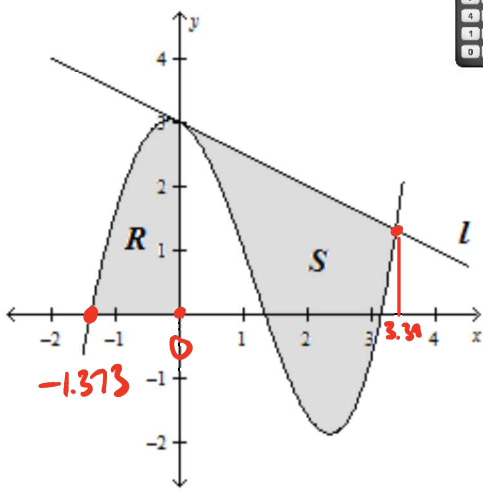
$$= -\frac{1}{\pi}(\cos 2\pi - \cos 0) - (4 - 8)$$

$$= -\frac{1}{\pi}(1 - 1) + 4$$

$$= -\frac{1}{\pi}(0) + 4$$

$$= 4$$

Pictured to the right is the graph of $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$ and a line, l, which is tangent to $f(x)$ at the point (0, 3).



Find the area of Region R.

Area $R = \int_{-1.373}^{0} f(x)\,dx$

≈ 2.903

Find the equation of line l if it is tangent to the graph of $f(x)$ at (0, 3).

PoT $= (0,3)$

SoT $= f'(0) = -\frac{1}{2}$

$y - 3 = -\frac{1}{2}(x - 0)$

$y - 3 = -\frac{1}{2}x$

$y = -\frac{1}{2}x + 3$

At what ordered pair, other than (0, 3), does the graph of line l intersect the graph of $f(x)$?

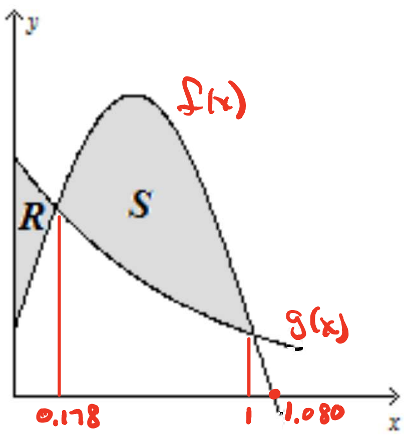
$(3.39, 1.305)$

Find the area of Region S.

Area $S = \int_{0}^{3.39} \left[\left(-\frac{1}{2}x + 3\right) - f(x)\right] dx$

$= 6.982$

Pictured are regions R and S, which are formed by the graphs of $f(x) = \frac{1}{4} + sin(\pi x)$ and $g(x) = 4^{-x}$



Identify the points of intersection of $f(x)$ and $g(x)$.

$(0.178, 0.781)$

$(1, 0.25)$

Find the area of Region *R*.

$$\text{Area } R = \int_0^{0.178} [g(x) - f(x)]\,dx$$

$$= 0.065$$

Find the area of Region *S*.

$$\text{Area } S = \int_{178}^{1} [f(x) - g(x)]\,dx$$

$$= 0.410$$

Find the area of the unshaded region bounded by the graphs of *f*, *g*, and the *x* – axis.

$$\text{Area} = \int_0^{0.178} f(x)\,dx + \int_{0.178}^{1} g(x)\,dx + \int_1^{1.080} f(x)\,dx$$

$$= .093 + .383 + .010$$

$$= 0.486$$