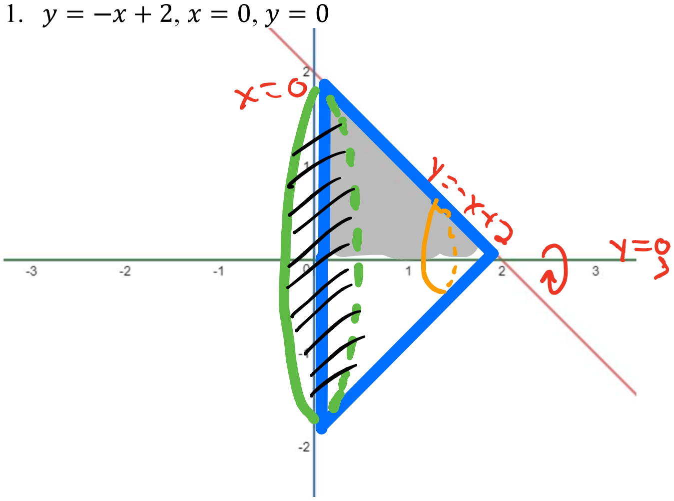
Name ____________________ **Date** __________ **Class** __________

Homework 7.6

Sketch the area bounded by the equations and revolve it around the x-axis. Find the volume of the resulting solid. Leave the answers in terms of π.

1. $y = -x + 2,\ x = 0,\ y = 0$



② $R(x) = -x+2$

$R^2 = x^2 - 4x + 4$

③ $D: [0,2]$

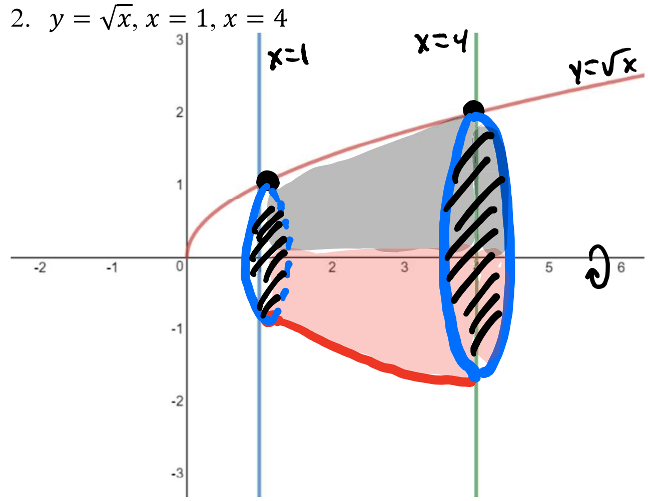
④ $V = \pi \int_a^b R^2(x)\,dx$

$V = \pi \int_0^2 [4 - 4x + x^2]\,dx$

$V = \pi\left(\frac{8}{3}\right)$ or 2.667π

$V = \frac{8}{3}\pi$ un^3

2. $y = \sqrt{x},\ x = 1,\ x = 4$



② $R(x) = \sqrt{x}$

$R^2 = x$

③ $D: [1,4]$

④ $V = \pi \int_a^b R^2(x)\,dx$

$V = \pi \int_1^4 (x)\,dx$

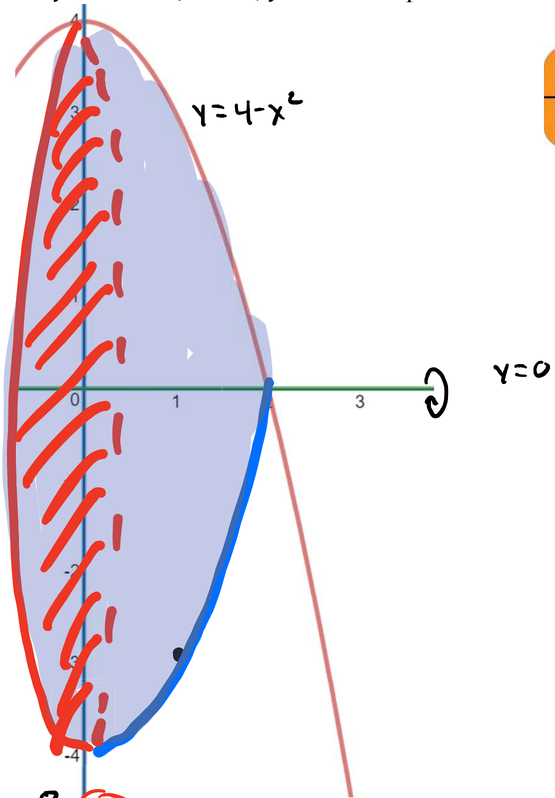
$V = \pi \frac{1}{2}x^2 \Big|_1^4$

$V = \left[\pi \frac{1}{2}(4)^2\right] - \left[\pi \frac{1}{2}(1)^2\right]$

$V = \left[\pi\left(\frac{1}{2}\right)16\right] - \left[\pi \frac{1}{2}\right]$

$V = \frac{15}{2}\pi$ un^3

3. $y = 4 - x^2$, $x = 0$, $y = 0$ quadrant I



② $R(x) = 4 - x^2$

$R^2 = 16 - 8x^2 + x^4$

③ $D: [0,2]$

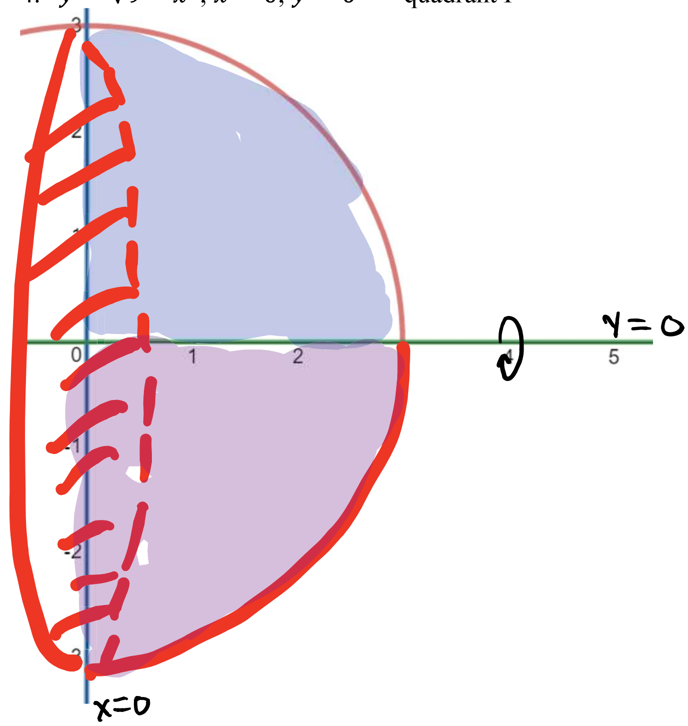
④ $V = \pi\int_a^b R^2(x)\,dx$

$V = \pi\int_0^2 [16 - 8x^2 + x^4]$

$V = \pi\left[\frac{256}{15}\right]$ or 17.067π

$V = \frac{256}{15}\pi$ un^3

4. $y = \sqrt{9 - x^2}$, $x = 0$, $y = 0$ quadrant I



② $R(x) = \sqrt{9 - x^2}$

$R^2 = 9 - x^2$

③ $D = [0,3]$

④ $V = \pi\int_a^b R^2(x)\,dx$

$V = \pi\int_0^3 [9 - x^2]\,dx$

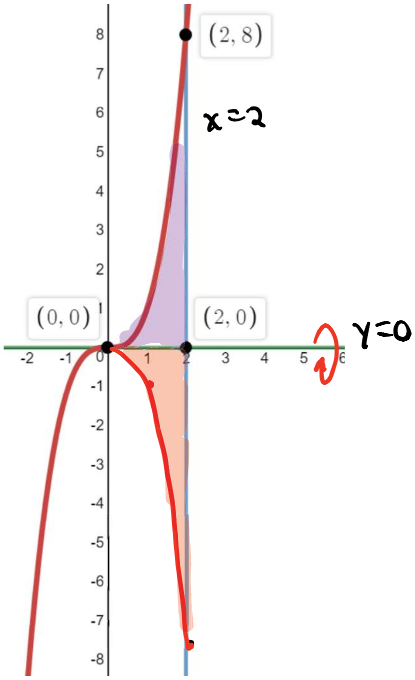
$V = \pi\left[9x - \frac{1}{3}x^3\right]\Big|_0^3$

$V = \pi\left[9(3) - \frac{1}{3}(3)^3\right] - \pi\left[9(0) - \frac{1}{3}(0)^3\right]$

$V = \pi[27 - 9] - \pi[0]$

$V = 18\pi$ un^3

5. $y = x^3, x = 2, y = 0$



② $R(x) = x^3$

$R^2 = x^6$

③ $D = [0, 2]$

④ $V = \pi \int_a^b R^2(x)\, dx$

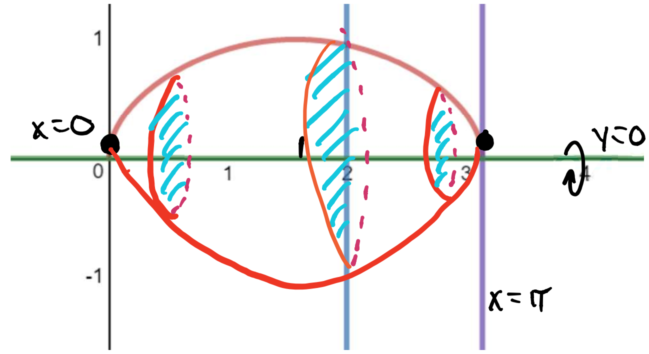
$V = \pi \int_0^2 x^6\, dx$

$V = \pi \frac{1}{7} x^7 \Big|_0^2$

$V = \left[\pi \frac{1}{7}(2)^7\right] - \left[\pi \frac{1}{7}(0)^7\right]$

$V = \frac{128}{7}\pi \text{ un}^3$

6. $y = \sqrt{\sin x}, x = 0, x = \pi, y = 0$



② $R(x) = \sqrt{\sin x}$

$R^2 = \sin x$

③ $D = [0, \pi]$

④ $V = \pi \int_a^b R^2(x)\, dx$

$V = \pi \int_0^\pi \sin x\, dx$

$V = \pi(-\cos x) \Big|_0^\pi$

$V = [-\pi \cos \pi] - [-\pi \cos 0]$

$V = [\pi(-1)] - [-\pi(1)]$

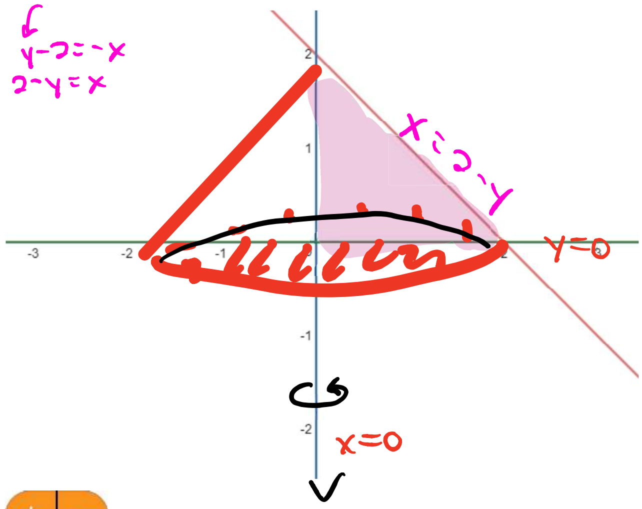
$V = \pi + \pi$

$V = 2\pi \text{ un}^3$

Sketch the area bounded by the equations and revolve it around the y-axis. Find the volume of the resulting solid. Leave the answers in terms of π.

7. $y = -x + 2$, $x = 0$, $y = 0$

$y - 2 = -x$
$2 - y = x$



(2) $R(y) = -y + 2$

$R^2 = y^2 - 4y + 4$

(3) $R: [0, 2]$

(4) $V = \pi \int_a^b R^2(y)\,dy$

$V = \pi \int_0^2 [4 - 4y + y^2]\,dy$

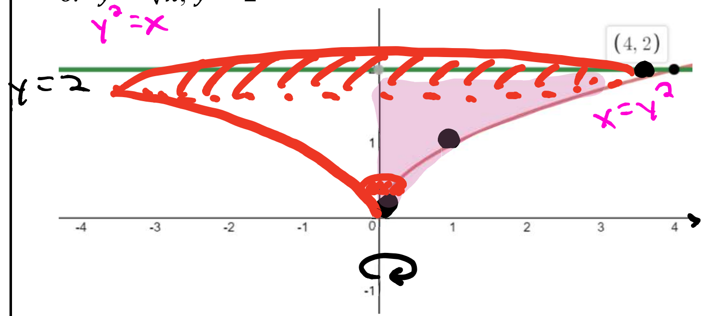
$V = \pi \left[8/3 \right]$ or 2.667π

$V = 8/3\,\pi\ un^3$

Compare to # 1

8. $y = \sqrt{x}$, $y = 2$

$y^2 = x$



(2) $R(y) = y^2$

$R^2 = y^4$

(3) $R: [0, 2]$

(4) $V = \pi \int_a^b R^2(y)\,dy$

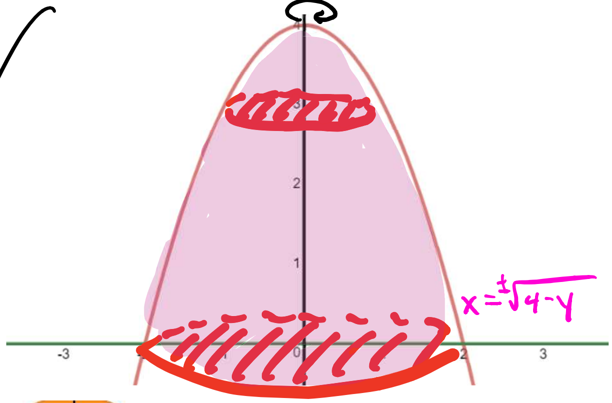
$V = \pi \int_0^2 y^4\,dy$

$V = \pi \left[\frac{32}{5} \right]$ or 6.4π

$V = \frac{32}{5}\pi\ un^3$

Compare to #2

9. $y = 4 - x^2$, $x = 0$, $y = 0$



$y - 4 = -x^2$

$4 - y = x^2$

$\pm\sqrt{4-y} = x$

only need + b/c its revolved

② $R(y) = \sqrt{4-y}$

$R^2 = 4 - y$

③ $R: [0, 4]$

④ $V = \pi \int_a^b R^2(y)\, dy$

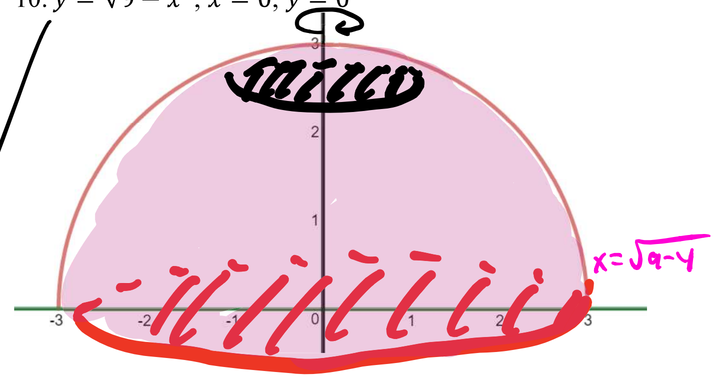
$V = \pi \int_0^4 (4-y)\, dy$

$V = \pi [8]$

$V = 8\pi$ un^3

Compare to # 3

10. $y = \sqrt{9 - x^2}$, $x = 0$, $y = 0$



$y^2 = 9 - x^2$

$y^2 - 9 = -x^2$

$9 - y^2 = x^2$

$\pm\sqrt{9-y^2} = x$

only need + b/c its revolved

② $R(y) = \sqrt{9-y^2}$

$R^2 = 9 - y^2$

③ $R: [0, 3]$

④ $V = \pi \int_a^b R^2(y)\, dy$

$V = \pi \int_0^3 (9-y^2)\, dy$

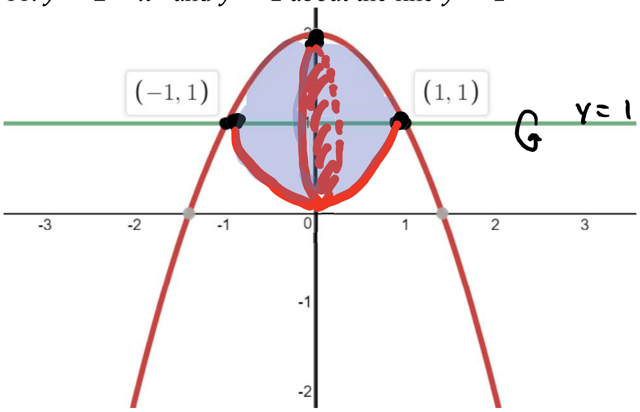
$V = \pi [18]$

$V = 18\pi$ un^3

Compare to # 4

Sketch the area bounded by the equations and revolve it around the given line. Find the volume of the resulting solid. Leave the answers in terms of π.

11. $y = 2 - x^2$ and $y = 1$ about the line $y = 1$



② $R(x) = \text{Top} - \text{Bottom} = (2-x^2) - (1) = 1 - x^2$

$R^2 = 1 - 2x^2 + x^4$

③ $D: [-1, 1]$

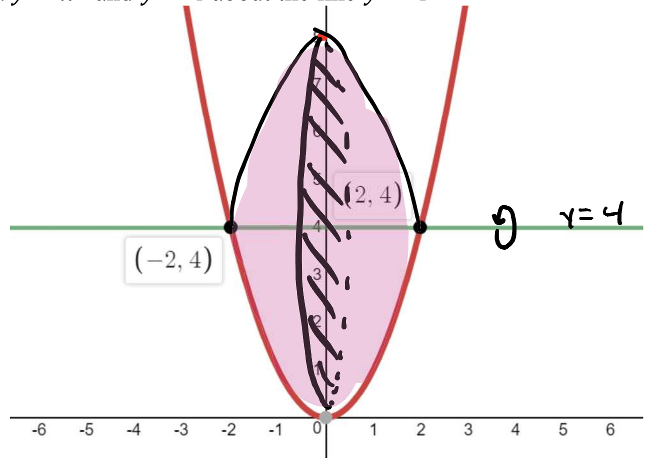
④ $V = \pi \int_a^b [R(x)]^2 dx$

$V = \pi \int_{-1}^{1} [1 - 2x^2 + x^4] dx$

$V = \pi \left[\frac{16}{15}\right]$ or 1.067

$V = \frac{16}{15}\pi$ un^3

12. $y = x^2$ and $y = 4$ about the line $y = 4$



② $R(x) = \text{Top} - \text{Bottom} = (4) - (x^2) = 4 - x^2$

$R^2 = 16 - 8x^2 + x^4$

③ $D: [-2, 2]$

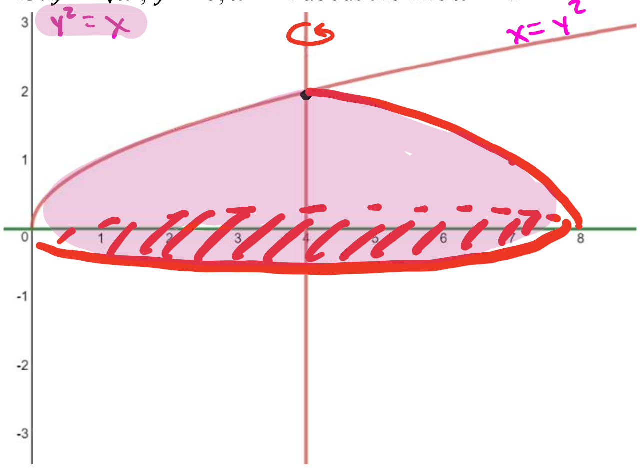
④ $V = \pi \int_a^b R^2(x)\, dx$

$V = \pi \int_{-2}^{2} [16 - 8x^2 + x^4] dx$

$V = \pi \left[\frac{512}{15}\right]$ or 34.133π

$V = \frac{512}{15}\pi$ un^3

13. $y = \sqrt{x}$, $y = 0$, $x = 4$ about the line $x = 4$



(2) $R(y) = \text{RIGHT} - \text{LEFT} = (4) - (y^2) = 4 - y^2$

$R^2 = 16 - 8y^2 + y^4$

(3) $R: [0, 2]$

(4) $V = \pi \int_a^b R^2(y)\,dy$

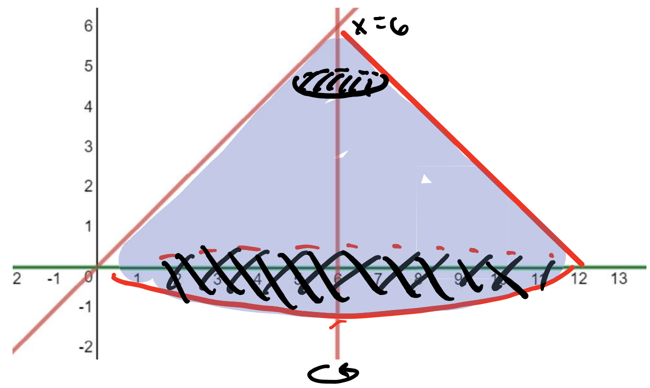
$V = \pi \int_0^2 [16 - 8y^2 + y^4]\,dy$

$V = \pi \left[\frac{256}{15}\right]$ or 17.067π

$V = \frac{256}{15}\pi$ un^3

Compare to #12

14. $y = x$, $y = 0$, $x = 6$ about the line $x = 6$



(2) $R(y) = \text{RIGHT} - \text{LEFT} = (6) - (y) = 6 - y$

$R^2 = 36 - 12y + y^2$

(3) $R: [0, 6]$

(4) $V = \pi \int_a^b R^2(y)\,dy$

$V = \pi \int_0^6 [36 - 12y + y^2]\,dy$

$V = \pi [72]$

$V = 72\pi$ un^3

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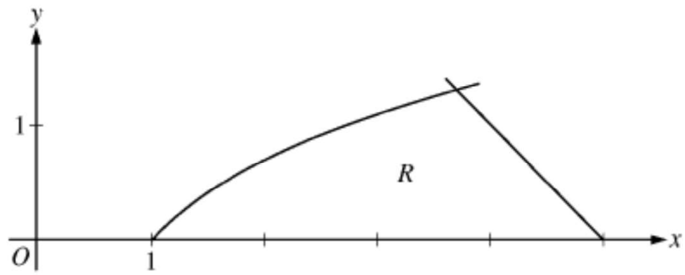
Question 2



Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.
