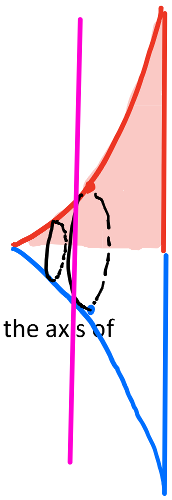
7.6 – Volumes of Solids of Revolution

Volume of a Solid of Revolution Disc

$$V = \pi \int_a^b [R(x)]^2 dx$$

Or

$$V = \pi \int_a^b [R(y)]^2 dy$$



Where R(x) or R(y) is the distance between the axis of revolution and the outside of the object.

1) Graph all boundaries

2) Find $R^2(x)$
   a. If axis of rotation is x-axis, $R(x) = f(x)$
   b. If axis of rotation is $y = a$, $R(x) = |f(x) - a|$

OR

Find $R^2(y)$
   a. If axis of rotation is y-axis, R(y)=f(y)
   b. If axis of rotation is $x = b$, $R(y) = |f(y) - b|$

3) Find b (upper bound) and a (lower bound) from visual inspection of graph or by setting boundaries equal to each other.

4) Find Volume

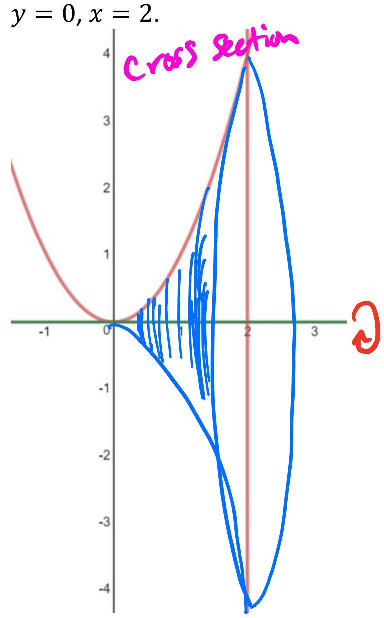
$$Volume = \pi \int_{x=a}^{x=b} [R(x)]^2 dx$$

or

$$Volume = \pi \int_{y=a}^{y=b} [R(y)]^2 dy$$

Finding the volume of a solid of revolution.

1. Sketch the area bounded by the equations. $y = x^2$, $y = 0$, $x = 2$.



2. Revolve it around the *x*-axis to create a solid.

3. What does the area of a cross section look like?

Circle

4. What is the area of a circle?

$A = \pi r^2$

5. What is the radius of this circle? $f(x)$

Instead of $f(x)$, we will call it $R(x)$.

6. What is the area of one cross-section?

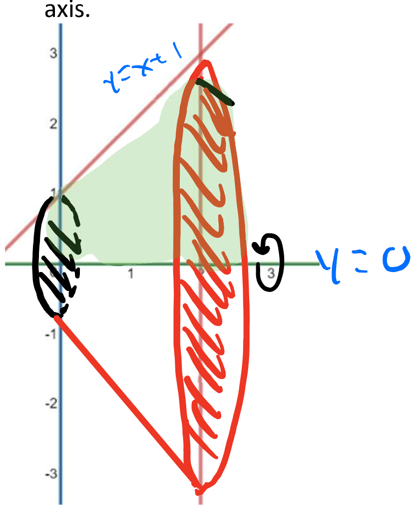
$\pi [x^2]^2$ ← RADIUS

7. What is the volume of the solid?

$V = \int_0^2 \pi [x^2]^2 dx$

Find the volume of the solid formed by revolving the given boundaries about the given axis of rotation.

1. $y = x + 1, y = 0, x = 0, x = 2.$ Revolve about the x-axis.



② $R(x) = x+1$

$R^2 = x^2+2x+1$

③ D: $[0,2]$

④ $V = \pi \int_{x=a}^{x=b} R^2(x)\,dx$

$V = \pi \int_0^2 (x^2+2x+1)\,dx$

$V = \pi \left[\frac{1}{3}x^3+x^2+x\right]\Big|_0^2$

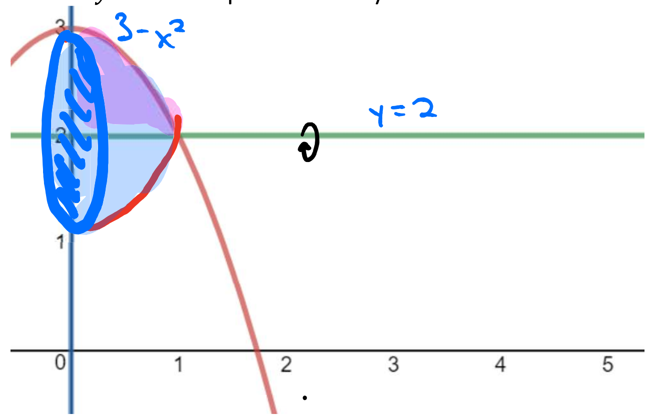
$= \pi\left[\frac{1}{3}(2)^3+(2)^2+(2)\right] - \pi\left[\frac{1}{3}(0)^3+(0)^2+(0)\right]$

$= \pi\left[\frac{8}{3}+4+2\right] - \pi[0]$

$= \pi\left[\frac{8}{3}+\frac{12}{3}+\frac{6}{3}\right]$

$V = \frac{26}{3}\pi \text{ un}^3$

2. $y = 3 - x^2, y = 2, x = 0, x = 1.$ Revolve about the line $y = 2$. Use quadrant I only



② $R(x) = (3-x^2)-(2) = -x^2+1$

$R^2 = x^4-2x^2+1$

③ Cross? Not on $(0,1)$; $D=[0,1]$

$3-x^2 = 2$

$1 = x^2$

$\pm 1 = x$

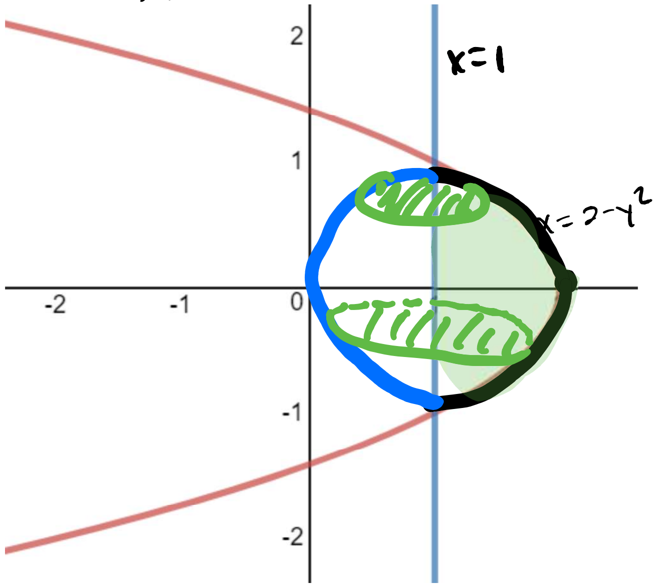
④ $V = \pi \int_{x=a}^{x=b} R^2(x)\,dx$

$V = \pi \int_0^1 (x^4-2x^2+1)\,dx$

$V = \pi \frac{8}{15}$ or 0.533π

$V = \frac{8}{15}\pi \text{ un}^3$

3. $x = 2 - y^2, x = 1$. Revolve about the line $x = 1$.



② $R(x) = (2-y^2) - (1) = -y^2 + 1$

$R^2 = y^4 - 2y^2 + 1$

③ Cross? D: $[-1,1]$

$2 - y^2 = 1$
$1 = y^2$
$\pm 1 = y$

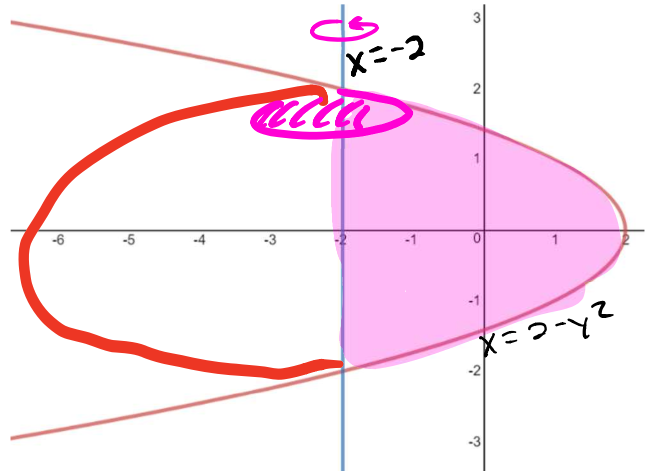
④ $V = \pi \int_{y=a}^{y=b} R(y)^2 \, dy$

$V = \pi \int_{-1}^{1} (y^4 - 2y^2 + 1) dy$

$V = \pi \frac{16}{15}$ or 1.067π

$V = \frac{16}{15}\pi \text{ un}^3$

4. $x = 2 - y^2, x = -2$. Revolve about the line $x = -2$.



② $R(y) = (2-y^2) - (-2) = -y^2 + 4$

$R^2 = y^4 - 8y^2 + 16$

③ Cross? D: $[-2,2]$

$2 - y^2 = -2$
$4 = y^2$
$\pm 2 = y$

④ $V = \pi \int_{y=a}^{y=b} R(y)^2 \, dy$

$= \pi \int_{-2}^{2} (y^4 - 8y + 16) dy$

$= \pi \frac{384}{5}$ or 76.8π

$V = \frac{384}{5}\pi \text{ un}^3$