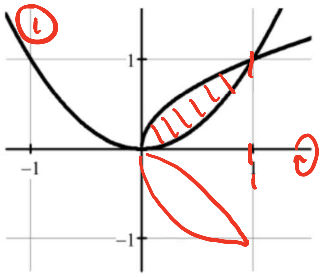
7.7 – **Volume of a Solid of Revolution Washers**

$$V = \pi \int_a^b [R^2(x) - r^2(x)]dx$$

Where R(x) is the radius of the outer function and r(x) is the radius of the inner function.

1. Find the volume if the region enclosed by $y = \sqrt{x}$ and $y = x^2$ is rotated about the x-axis.



② $R^2 = (\sqrt{x})^2 = x$
$r^2 = (x^2)^2 = x^4$

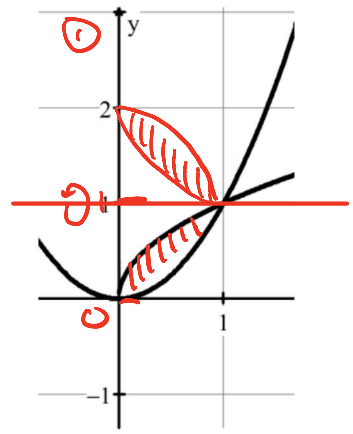
③ $D: [0, 1]$

④ $V = \pi \int_a^b [R^2 - r^2] dx$

$V = \pi \int_0^1 [x - x^4] dx$

$V = \frac{3}{10}\pi \text{ un}^3$

2. Find the volume if the region enclosed by $y = \sqrt{x}$ and $y = x^2$ is rotated about the line $y = 1$.



② $R^2 = (1 - x^2)^2$
$r^2 = (1 - \sqrt{x})^2$

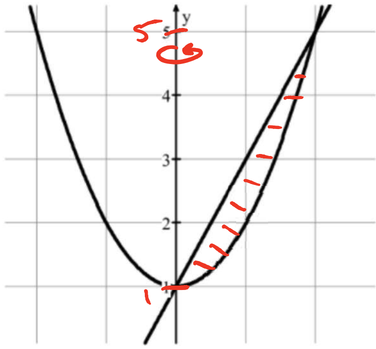
③ $R: [0, 1]$

④ $V = \pi \int_a^b [R^2 - r^2] dx$

$V = \pi \int_0^1 [(1-x^2)^2 - (1-\sqrt{x})^2] d$

$V = .367\pi \text{ un}^3$

3. Find the volume if the region enclosed by $y = x^2 + 1$ and $y = 2x + 1$ is rotated about the y-axis



$y = x^2 + 1$ | $y = 2x + 1$

$y - 1 = x^2$ | $y - 1 = 2x$

$\pm\sqrt{y-1} = x$ | $\frac{1}{2}y - \frac{1}{2} = x$

② $R^2 = \left(\sqrt{y-1}\right)^2 = y - 1$

$r^2 = \left(\frac{1}{2}y - \frac{1}{2}\right)^2$

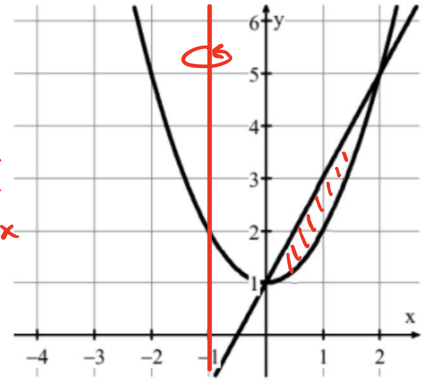
③ $R: [1, 5]$

④ $V = \pi\int_a^b \left[R^2 - r^2\right] dy$

$V = \pi \int_1^5 \left[(y-1) - \left(\frac{1}{2}y - \frac{1}{2}\right)^2\right] dy$

$V = 2.667\pi$ un^3

4. Find the volume if the region enclosed by $y = x^2 + 1$ and $y = 2x + 1$ is rotated about the line $x = -1$.



$y = 2x + 1$ | $y = x^2 + 1$

$y - 1 = 2x$ | $y - 1 = x^2$

$\frac{1}{2}y - \frac{1}{2} = x$ | $\pm\sqrt{y-1} = x$

② $R^2 = \left(\sqrt{y-1} + 1\right)^2$

$r^2 = \left[\left(\frac{1}{2}y - \frac{1}{2}\right) + 1\right]^2 = \left(\frac{1}{2}y + \frac{1}{2}\right)^2$

③ $R: [1, 5]$

④ $V = \pi\int_a^b \left[R^2 - r^2\right] dy$

$V = \pi\int_1^5 \left[\left(\sqrt{y-1} + 1\right)^2 - \left(\frac{1}{2}y + \frac{1}{2}\right)^2\right] dy$

$V = 5.333\pi$ un^3