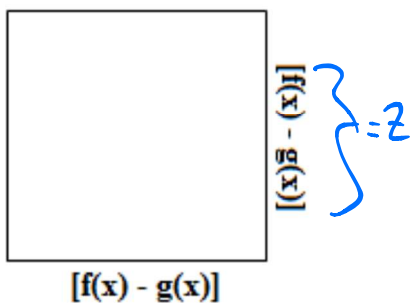
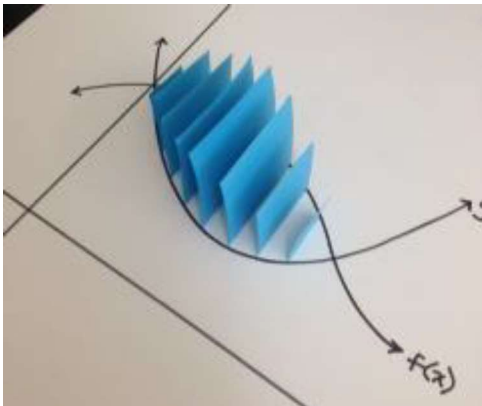


Notes 7.8 – Volumes of Solids with Known Cross Sections

Calculus cannot only be used to find the volume of solids created by revolving two-dimensional shapes around an axis but also the volume of solids formed by cross sections that are geometric shapes. In this lesson, you will derive the formulas for finding volumes of solids given that their cross sections are squares, isosceles right triangles, equilateral triangles, and semi circles.

Cross Sections that are Squares



Find the area of the square above in terms of $f(x)$ and $g(x)$.

$$A = b \cdot h$$

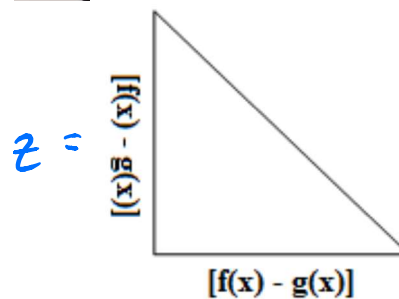
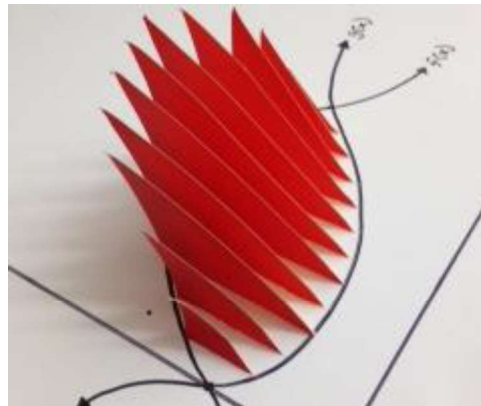
$$A = [f(x) - g(x)] [f(x) - g(x)]$$

$$A = [f(x) - g(x)]^2$$

If all of the cross sectional areas were added up, the total would be the volume of the solid pictured. How do we write this in calculus using an integral?

$$V = \int_a^b [f(x) - g(x)]^2 dx$$

Cross Sections that are Isosceles Right Triangles



Find the area of the triangle above in terms of $f(x)$ and $g(x)$.

$$A = \frac{1}{2} bh$$

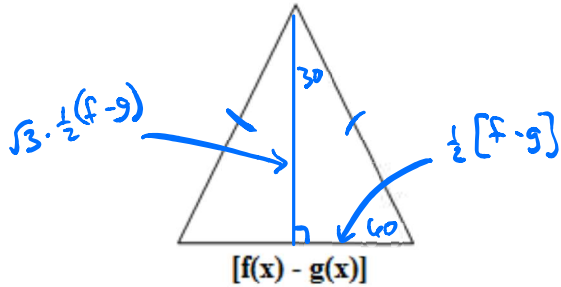
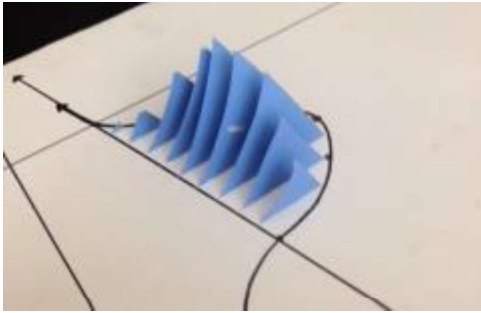
$$A = \frac{1}{2} [f(x) - g(x)] [f(x) - g(x)]$$

$$A = \frac{1}{2} [f(x) - g(x)]^2$$

If all of the cross sectional areas were added up, the total would be the volume of the solid pictured. How do we write this in calculus using an integral?

$$V = \frac{1}{2} \int_a^b [f(x) - g(x)]^2 dx$$

Cross Sections that are Equilateral Triangles



Find the area of the triangle above in terms of $f(x)$ and $g(x)$.

$$A_{\Delta} = \frac{1}{2} b \cdot h$$

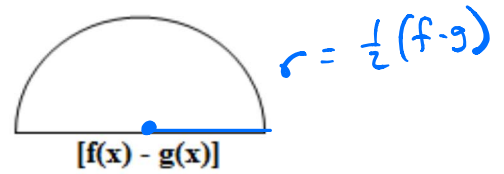
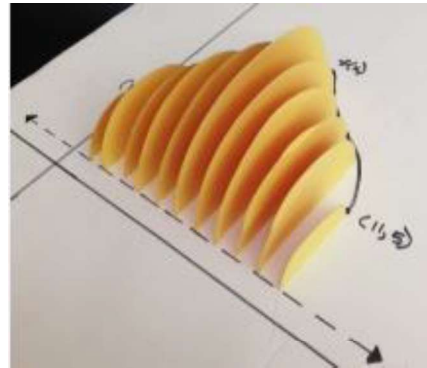
$$= \frac{1}{2} (f-g) \cdot \frac{\sqrt{3}}{2} (f-g)$$

$$A_{\Delta} = \frac{\sqrt{3}}{4} [f(x) - g(x)]^2$$

If all of the cross sectional areas were added up, the total would be the volume of the solid pictured. How do we write this in calculus using an integral?

$$V = \frac{\sqrt{3}}{4} \int_a^b [f(x) - g(x)]^2 dx$$

Cross Sections that are Semicircles



Find the area of the semicircle above in terms of $f(x)$ and $g(x)$.

$$A_{\circ} = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \pi \left(\frac{1}{2} (f-g) \right)^2$$

$$= \frac{1}{2} \pi \cdot \frac{1}{4} (f-g)^2$$

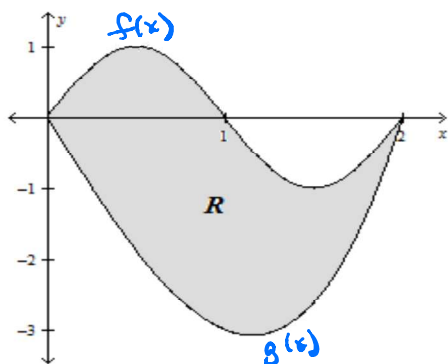
$$A_{\circ} = \frac{\pi}{8} [f(x) - g(x)]^2$$

If all of the cross sectional areas were added up, the total would be the volume of the solid pictured. How do we write this in calculus using an integral?

$$V = \frac{\pi}{8} \int_a^b [f(x) - g(x)]^2 dx$$

What do you notice about the integral-defined formulas for finding the volume of solids with certain cross sections?

R is bounded by $f(x) = \sin(\pi x)$ and $g(x) = x^3 - 4x$.



Find the volume of the solids formed whose cross sections are the shapes indicated below. The cross sections are perpendicular to the x -axis.

a. Cross sections are equilateral triangles

$$V = \frac{\sqrt{3}}{4} \int_0^2 [f(x) - g(x)]^2 dx$$

$$V = 4.321$$

b. Cross sections are semi-circles

$$V = \frac{\pi}{8} \int_0^2 [f(x) - g(x)]^2 dx$$

$$V = 3.918$$

c. Cross sections are isosceles right triangles

$$V = \frac{1}{2} \int_0^2 [f(x) - g(x)]^2 dx$$

$$V = 4.989$$

d. Cross sections are squares.

$$V = \int_0^2 [f(x) - g(x)]^2 dx$$

$$V = 9.978$$

e. Cross sections are rectangles whose height is twice the length of the base.

$$V = 2 \int_0^2 [f(x) - g(x)]^2 dx$$

$$V = 19.957$$

f. Cross sections are rectangles whose height is one-third the length of the base.

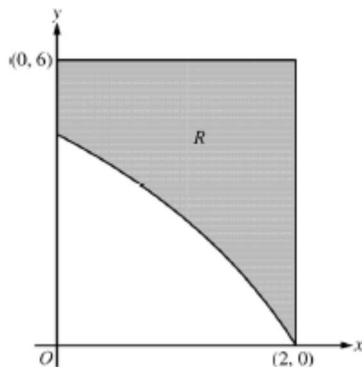
$$V = \frac{1}{3} \int_0^2 [f(x) - g(x)]^2 dx$$

$$V = 3.326$$

Summary

CALCULATOR PERMITTED
2010 AP© Calculus AB (Form B)
Question 1

In the figure, R is the shaded region in the first quadrant bounded by the graph of $y = 4 \ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.



A) Find the area of R

$$\text{Area} = \int_0^2 [6 - 4 \ln(3-x)] dx$$

$$\text{Area} = 6.817$$

B) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

$$V = \pi \int_0^2 [(8 - 4 \ln(3-x))^2 - (8-6)^2] dx$$

$$= 53.533 \pi$$

$$= 168.179$$

C) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

$$V = \int_0^2 [6 - 4 \ln(3-x)]^2 dx$$

$$V = 26.267$$