APC - Integration

8.1 – Definite Integrals

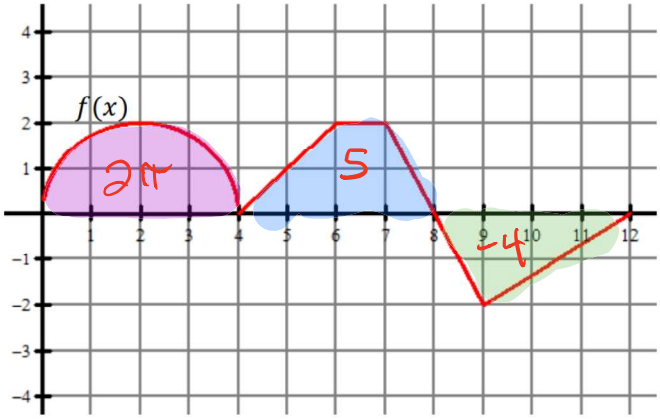
Basic Area Formulas

$A_{circle} = \pi r^2$

$A_{triangle} = \frac{1}{2}bh$

$A_{trap} = h\frac{1}{2}(b_1 + b_2)$

$A_{parallelogram} = bh$



$\int_0^4 f(x)\,dx =$ 2π

$\int_4^8 f(x)\,dx =$ 5

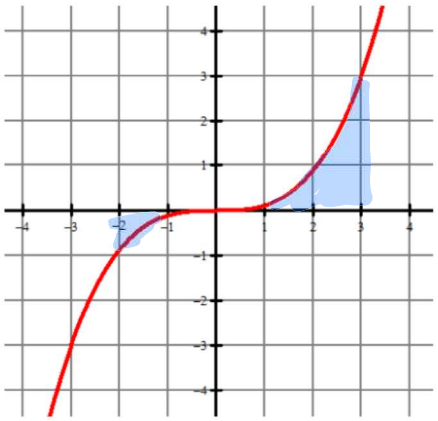
$\int_8^{12} f(x)\,dx =$ -4

$\int_0^8 f(x)\,dx =$ $2\pi + 5$

$\int_0^{12} f(x)\,dx =$ $2\pi + 5 - 4$

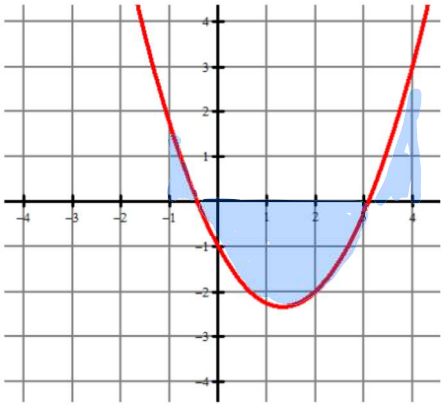
$= 2\pi + 1$

Write the integral for the area under $f(x) = \frac{1}{9}x^3$ from -2 to 3.



$\int_{-2}^{3} \frac{1}{9}x^3\,dx$

Write the integral for the area under $f(x) = \frac{3}{4}x^2 - 2x - 1$ from -1 to 4.



$\int_{-1}^{4} f(x)\,dx$

Write the integral for the area under $f(x) = x^2 + 1$ after you see it shaded.

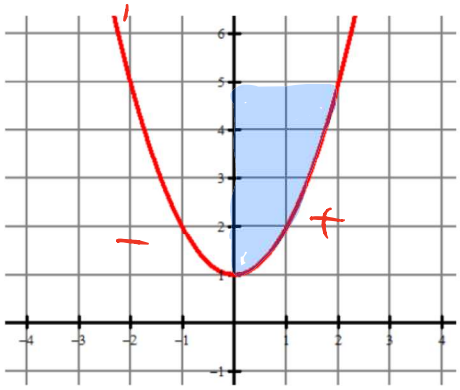
$y = x^2 + 1$

$y - 1 = x^2$

$\pm\sqrt{y-1} = x^2$

$\sqrt{y-1} = x$

$\int_1^5 \sqrt{y-1}\,dy$



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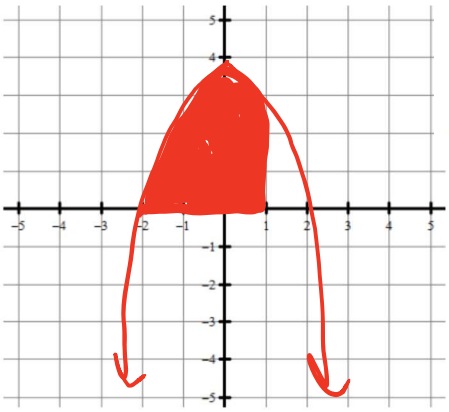
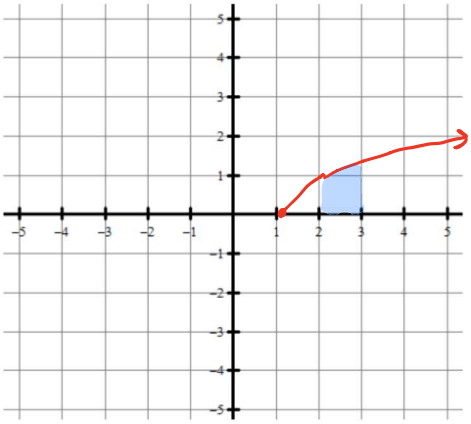
8.1 – Definite Integrals

Sketch a graph of the definite integral. Use a calculator to integrate.



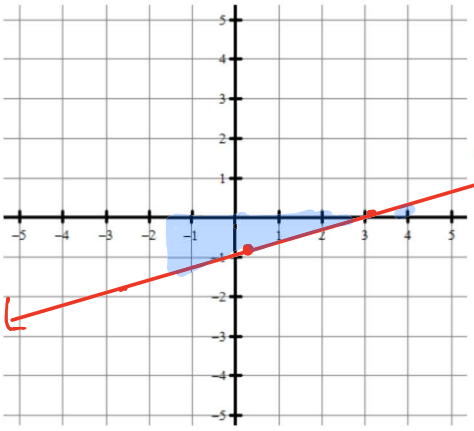
$$\int_{-2}^{1} (4 - x^2)\,dx$$

$= 9$



$$\int_{2}^{3} \sqrt{x-1}\,dx$$

≈ 1.218



$$\int_{-2}^{4} \left(\frac{x}{3} - 1\right) dx$$

≈ -4

Properties of Definite Integrals

Zero Integral

$$\int_{a}^{a} f(x)\,dx = 0$$

Negation

$$\int_{b}^{a} f(x)\,dx = -\int_{a}^{b} f(x)\,dx$$

Constant Multiple

$$\int_{b}^{a} k \cdot f(x)\,dx = k\int_{b}^{a} f(x)\,dx$$

Decomposition

$$\int_{a}^{b} f(x)\,dx = \int_{a}^{c} f(x)\,dx + \int_{c}^{b} f(x)\,dx$$

Assuming $a < c < b$

Addition/Subtraction

$$\int_{a}^{b} [f(x) \pm g(x)]\,dx = \int_{a}^{b} f(x)\,dx \pm \int_{a}^{b} g(x)\,dx$$

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Given the facts below, answer each question if possible.

$\int_{-2}^{1} f(x)dx = 4$ $\int_{1}^{5} f(x)dx = -3$ $\int_{-2}^{1} g(x)dx = 8$

(a) $\int_{5}^{1} f(x)dx = -\int_{1}^{5} f(x)dx$

$= -(-3)$

$= 3$

(d) $\int_{0}^{1} f(x)dx$ = not possible

(b) $\int_{-2}^{5} f(x)dx = \int_{-2}^{1} f(x)dx + \int_{1}^{5} f(x)dx$

$= 4 + (-3)$

$= 1$

(e) $\int_{1}^{-2} 3f(x)dx = -3\int_{-2}^{1} f(x)dx$

$= -3(4)$

$= -12$

(c) $\int_{-2}^{1} [f(x) + 2g(x)]dx = \int_{-2}^{1} f(x)dx + 2\int_{-2}^{1} g(x)dx$

$= 4 + 2(8)$

$= 4 + 16$

$= 20$

(f) $\int_{5}^{5} [f(x) - g(x)]dx = 0$