

# Trig Inverses

## Inverse Trig Derivatives

$$\frac{d}{dx} \sin^{-1}(u) = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} \sec^{-1}(u) = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} \tan^{-1}(u) = \frac{u'}{u^2+1}$$

- Assume  $u$  is a differentiable function.

Find the derivative.

$$\#1) \frac{d}{dx} \sin^{-1}(3x) = \frac{u'}{\sqrt{1-u^2}}$$

$$u = 3x$$

$$= \frac{3}{\sqrt{1-(3x)^2}}$$

$$\#2) \frac{d}{dx} \tan^{-1}(2x^2) = \frac{u'}{u^2+1}$$

$$u = 2x^2$$

$$= \frac{4x}{(2x^2)^2+1}$$

## Inverse Trig Derivatives

$$\frac{d}{dx} \cos^{-1}(u) = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} \csc^{-1}(u) = -\frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} \cot^{-1}(u) = -\frac{u'}{u^2+1}$$

- Assume  $u$  is a differentiable function.

Find the derivative.

$$\#3) \frac{d}{dx} \cos^{-1}(-10x) = -\frac{u'}{\sqrt{1-u^2}}$$

$$u = -10x$$

$$= -\frac{(-10)}{\sqrt{1-(-10x)^2}}$$

$$= \frac{10}{\sqrt{1-100x^2}}$$

$$\#4) \frac{d}{dx} \csc^{-1}(5x^3) = \frac{15x^2}{|5x^3|\sqrt{(5x^3)^2-1}}$$

$i = \text{inversed}$   
 $s = \sqrt{\phantom{x}}$   
 $c = \text{both crap}$   
 $\alpha = \text{add'n}$

$$\frac{d}{dx} \sin^{-1}(u) = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} \sec^{-1}(u) = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} \tan^{-1}(u) = \frac{u'}{u^2+1}$$

$$\frac{d}{dx} \cos^{-1}(u) = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} \csc^{-1}(u) = -\frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} \cot^{-1}(u) = -\frac{u'}{u^2+1}$$

### Trig Integrals:

$$\int \cos x \, dx = \sin(x) + C$$

$$\int -\csc x \cot x \, dx = \csc(x) + C$$

$$\int \sin x \, dx = -\cos(x) + C$$

$$\int \sec x \tan x \, dx = \sec(x) + C$$

$$\int \sec^2 x \, dx = \tan(x) + C$$

$$\int \csc^2 x \, dx = -\cot(x) + C$$

Preparing for u-substitution:

$$\int \cos \underbrace{ax}_{(\text{linear})} \, dx = \frac{1}{a} \sin(x) + C$$

$$1. \int \cos(4x) \, dx = \frac{1}{4} \sin(4x) + C$$

$$3. \int \sec(3x) \tan(3x) \, dx = \frac{1}{3} \sec(3x) + C$$

$$2. \int \sec^2\left(\frac{1}{2}x\right) \, dx = 2 \ln\left(\frac{1}{2}x\right) + C$$

$$4. \int \csc\left(\frac{1}{2}x\right) \cot\left(\frac{1}{2}x\right) \, dx = -2 \csc\left(\frac{1}{2}x\right) + C$$

## Integration involving inverse trig functions

$$\int \frac{u'}{\sqrt{1-u^2}} du = \sin^{-1}(u) + C$$

$$\int \frac{u'}{|u|\sqrt{u^2-1}} du = \sec^{-1}(u) + C$$

$$\int \frac{u'}{u^2+1} du = \tan^{-1}(u) + C$$

$$\int -\frac{u'}{\sqrt{1-u^2}} du = \cos^{-1}(u) + C$$

$$\int -\frac{u'}{|u|\sqrt{u^2-1}} du = \csc^{-1}(u) + C$$

$$\int -\frac{u'}{u^2+1} du = \cot^{-1}(u) + C$$

Integrate.

$$5. \int -\frac{1}{\sqrt{1-x^2}} dx = \int -\frac{u'}{\sqrt{1-u^2}} du = \cos^{-1}(x) + C$$

$$u^2 = x^2$$

$$u = x$$

$$u' = 1$$

$$7. \int \frac{3}{9x^2+1} dx = \int \frac{3}{(3x)^2+1} dx$$

$$u^2 = 9x^2 \quad = \int \frac{u'}{u^2+1} du$$

$$u = 3x$$

$$u' = 3$$

$$= \tan^{-1}(3x) + C$$

$$6. \frac{1}{2} \int -\frac{1 \cdot 2}{|2x|\sqrt{4x^2-1}} dx = \frac{1}{2} \int -\frac{u'}{|u|\sqrt{u^2-1}} du$$

$$u = 2x \quad = \frac{1}{2} \csc^{-1}(2x) + C$$

$$u^2 = 4x^2$$

$$u' = 2$$

$$8. \int \frac{20x^3}{\sqrt{1-25x^8}} dx = \int \frac{u'}{\sqrt{1-u^2}} du$$

$$u^2 = 25x^8 \quad = \sin^{-1}(5x^4) + C$$

$$u = 5x^4$$

$$u' = 20x^3$$

## Hw Trig Inverses

**Find the following.**

1. $\frac{d}{dx} \sin^{-1}(5x)$	2. $\frac{d}{dx} \csc^{-1}(4x^5)$	3. $\frac{d}{dx} \tan^{-1}(2x)$
4. $\frac{d}{dx} \frac{\sin x}{x}$	5. $\frac{d}{dx} \sec^{-1}(x^3)$	6. $\frac{d}{dx} \csc 6x$
7. $\lim_{x \rightarrow 2} \frac{x-2}{x^2+5x-14}$	8. $\frac{d}{dx} \cos^{-1}(3x^2)$	9. Anti-derivative of $f'(x) = \frac{5}{\sqrt{1-25x^2}}$

1. Compute the derivative of  $f(x) = \ln x - \sin x + \arctan x + 2^x, x > 0$ .

(A)  $f(x) = \frac{1}{x} - \cos x + \frac{1}{1+x^2} + x2^x$

(B)  $f(x) = \frac{1}{x} - \cos x + \frac{1}{1-x^2} + x2^x$

(C)  $f(x) = \frac{1}{x} + \cos x + \frac{1}{1-x^2} + (\ln 2)2^x$

(D)  $f(x) = \frac{1}{x} - \cos x + \frac{1}{1+x^2} + (\ln 2)2^x$

(E)  $f(x) = \frac{1}{x} + \cos x + \frac{1}{1+x^2} + (\ln 2)2^x$

2. What is an equation for the line tangent to  $y = \tan^{-1} x$  at  $x = \sqrt{3}$ ?

(A)  $y - \frac{\pi}{3} = -\frac{1}{2}(x - \sqrt{3})$

(B)  $y - \frac{\pi}{6} = -\frac{1}{4}(x - \sqrt{3})$

(C)  $y - \frac{\pi}{3} = -\frac{1}{4}(x - \sqrt{3})$

(D)  $y - \frac{\pi}{6} = \frac{3}{4}(x - \sqrt{3})$

(E)  $y - \frac{\pi}{3} = \frac{1}{4}(x - \sqrt{3})$

In this practice set you will find definite integrals, indefinite integrals, AND derivatives.

1.  $\int (\cos x - 5 \sin x) dx$

2.  $\int \sec x (\sec x + \tan x) dx$

3.  $\int_{\pi/4}^{\pi} 2 \cos x dx$

7.  $\int \frac{3}{|x|\sqrt{36x^6 - 1}} dx$

8.  $\int -\frac{2}{4x^2 + 1} dx$

9.  $\frac{d}{dx} \sin 5x$

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10. $\frac{d}{dx} \sec^2 2x$	11. $\int (\sec^2 x + x) dx$	12. $\int \frac{\sin x}{\cos^2 x} dx$
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15.  $\int \frac{20x^4}{\sqrt{1 - 16x^{10}}} dx$

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16. $\int \frac{\cos^3 x + 4}{\cos^2 x} dx$	17. $\int x - \frac{2}{\cos^2 x} dx$	18. $\int \frac{72x^7}{1 + 81x^8} dx$
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