Basic Integration

8.2 – 1st Fundamental Theorem of Integral Calculus

Antidifferentiation and Integration are synonyms. Everything a derivative does, antidifferentiation does backwards. By the way, marginal means derivative.

Find the antiderivative.

#1) $f'(x) = 2x^3 - 5x + 7$

$f(x) = \frac{1}{2}x^4 - \frac{5}{2}x^2 + 7x + C$

#2) $f'(x) = \sqrt{x} - 4 = x^{\frac{1}{2}} - 4$

$f(x) = \frac{2}{3}x^{3/2} - 4x + C$

Indefinite Integral

$$\int f(x)dx = F(x)$$

where $F'(x) = f(x)$

Power rule for Integration

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad (n \neq -1)$$

Evaluate the indefinite integrals

#1) $\int (9 - x^2)dx = 9x - \frac{1}{3}x^3 + C$

#2) $\int \left(\frac{6}{t^2} - t^{-3}\right) dt = \int (6t^{-2} - t^{-3})dt$

$= -6t^{-1} + \frac{1}{2}t^{-2} + C$

#3) $\int \left(\frac{2w^3 - 4w^2 + 7w}{w}\right) dw = \int (2w^2 - 4w + 7)dw$

$= \frac{2}{3}w^3 - 2w^2 + 7w + C$

1st Fundamental Theorem of Integral Calculus

$$\int_a^b f(x)dx = F\Big|_a^b = F(b) - F(a)$$

for a continuous function *f* on an interval [*a, b*], where F is any antiderivative of *f*.

Steps to Evaluate a Definite integral

#1: Find an *indefinite* integral of the function (omitting the + C)

#2: *Evaluate* the result at *b* and *subtract* the evaluation at *a*.

Properties of Definite Integrals

$$\int_a^b c \cdot f(x)dx = c\int_a^b f(x)dx$$

$$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

Net Accumulation at a Given Rate

$$Net\ Change = \int (rate)dx$$

Evaluate each definite integral.

#1) $\int_2^6 (3x^2 + x - 2)dx = \left[x^3 + \frac{1}{2}x^2 - 2x\right]\Big|_2^6$

$= \left[(6)^3 + \frac{1}{2}(6)^2 - 2(6)\right] - \left[(2)^3 + \frac{1}{2}(2)^2 - 2(2)\right]$

$= \left[216 + \frac{1}{2}(36) - 12\right] - \left[8 + \frac{1}{2}(4) - 4\right]$

$= [204 + 18] - [4 + 2]$

$= [222] - [6]$

$\int_2^6 (3x^2 + x - 2)dx = 216$

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#2) $\int_{-2}^{5}(4-6x)\,dx = \left[4x-3x^2\right]\Big|_{-2}^{5}$

$= [4(5)-3(5)^2]-[4(-2)-3(-2)^2]$

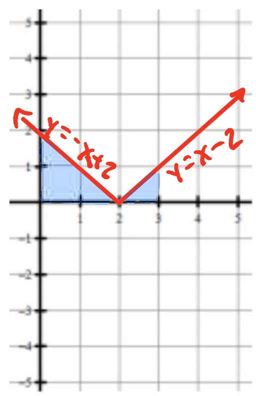
$= [20-3(25)]-[-8-3(4)]$

$= 20-75+8+12$

$\int_{-2}^{5}(4-6x)\,dx = -35$

#3) $\int_0^3 |x-2|\,dx$

$= \int_0^2(-x+2)\,dx + \int_2^3(x-2)\,dx$



$= \frac{1}{2}(2)(2) + \frac{1}{2}(1)(1)$

$= 2 + \frac{1}{2}$

$= 2.5$ un^2

#1) Find the area under $y = x^2$ from $x = 0$ to $x = 1$.

$\int_0^1 x^2\,dx = \left[\frac{1}{3}x^3\right]\Big|_0^1$

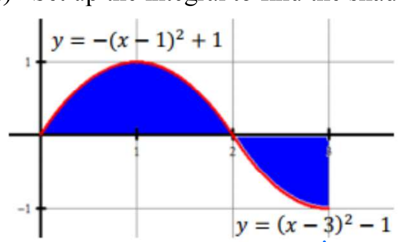
$= \left[\frac{1}{3}(1)^3\right] - \left[\frac{1}{3}(0)^3\right]$

$= \left[\frac{1}{3}(1)\right] - \left[\frac{1}{3}(0)\right]$

$= \frac{1}{3} - 0$

$\int_0^1 x^2\,dx = \frac{1}{3}$ un^2

#2) Set up the integral to find the shaded area.



$A = \int_0^2 [-(x-1)^2+1]\,dx + \int_2^3 [(x-3)^2-1]\,dx$

T&F Bombs

#1) While indulging his senses at Medieval Times, George is inspired to start yet another business. He conjectures his marginal cost function would be $MC(x) = \frac{50}{\sqrt{x}}$ where x is the number of tar and feather bombs produced. Find the total cost of making tar and feather bombs 100 to 400.

C = total cost

$C = \int_{100}^{400} 50x^{-\frac{1}{2}}\,dx$

$= 2(50)x^{\frac{1}{2}}\Big|_{100}^{400}$

$= 100\sqrt{x}\Big|_{100}^{400}$

$= [100\sqrt{400}] - [100\sqrt{100}]$

$= 100\cdot 20 - 100\cdot 10$

$= 2000 - 1000$

$C = 1000$

It cost \$1000 to make T&F bombs 100 to 400.

Poison Arrow Tips

#2) Trying to diversify, George decides to also manufacture poison arrow tips. Being hip to his own mortality, George enlists the aid of some local teen runaways as QATs, quality assurance technicians. A QAT can test poison arrow tips at the rate of $-3t^2 + 15t + 8$ tips per second (for $t \le 6$ because 6 seconds after licking the first tip, poison sets in and kills the technician), where t is the number of seconds after 9:00 A.M. How many tips can be tested between 9:00 and 1 second A.M. and 9:00 and 4 seconds A.M.?

T = tips

$T = \int_1^4 (-3t^2+15t+8)\,dt$

$= \left[-t^3 + \frac{15}{2}t^2 + 8t\right]\Big|_1^4$

$= \left[-(4)^3 + \frac{15}{2}(4)^2 + 8(4)\right] - \left[-(1)^3 + \frac{15}{2}(1) + 8(1)\right]$

$= \left[-64 + \frac{15}{2}(16) + 32\right] - \left[-1 + \frac{15}{2} + 8\right]$

$= [+120 - 32] - [7 + 7.5]$

$= [88] - [14.5]$

$T = 73.5$

QATs can test 73.5 arrows from 1 to 4 seconds after 9:00 Am