Basic Integration
8.3 – Antiderivatives

Find the antiderivative.

#1) $f'(x) = \sqrt[5]{x^2} + \frac{4}{\sqrt[4]{x^3}}$ $= x^{2/5} + 4x^{-3/4}$

$f(x) = \frac{5}{7}x^{7/5} + 4(4)x^{1/4}$

$f(x) = \frac{5}{7}\sqrt[5]{x^7} + 16\sqrt[4]{x}$

Recall…. each derivative

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\sin x) = \cos x$$

More Antiderivative formulas

$\int \frac{1}{x} dx = \ln|x| + C$ $\quad \int e^x = e^x + C$

$\int \cos x\, dx = \sin x + C$ $\quad \int \sin x\, dx = -\sin x + C$

You can calculate C if you're given a condition.

Find the function f that satisfies the given condition.

#1) $f'(x) = 3x^2 - x + 4$ and $f(2) = 8$

$f(x) = x^3 - \frac{1}{2}x^2 + 4x + C$

@(2,8): $8 = (2)^3 - \frac{1}{2}(2)^2 + 4(2) + C$

$8 = 8 - \frac{1}{2}(4) + 8 + C$

$8 = 16 - 2 + C$

$8 = 14 + C$

$-6 = C$

$\therefore f(x) = x^3 - \frac{1}{2}x^2 + 4x - 6$

#2) $f'(x) = \frac{1}{x} - e^x$ and $f(1) = e$

$f(x) = \ln|e| - e^x + C$

@(1,e): $e = \ln|1| - e^1 + C$

$e = 0 - e + C$

$2e = C$

$\therefore f(x) = \ln|e| - e^x + 2e$

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#3) Find y if $y'' = \cos x$ and $y'\left(\frac{\pi}{2}\right) = 2$ and $y\left(\frac{\pi}{2}\right) = 3\pi$

$y' = \sin x + C$

@$(\frac{\pi}{2}, 2)$: $2 = \sin\frac{\pi}{2} + C$

$2 = 1 + C$

$1 = C$

$\therefore y' = \sin x + 1$

$y = -\cos x + x + C$

@$(\frac{\pi}{2}, 3\pi)$ $3\pi = -\cos\frac{\pi}{2} + \frac{\pi}{2} + C$

$3\pi = 0 + \frac{\pi}{2} + C$

$\frac{6\pi}{2} = \frac{\pi}{2} + C$

$\frac{5\pi}{2} + C$

$\therefore y = -\cos x + x + \frac{5\pi}{2}$

Particle Motion

A particle moves along the x-axis with an acceleration of $a(t) = 12t - 4$. The particle's velocity is 18 centimeters per second at t = 2. The initial position of the particle is 8 cm. What is the position of the particle at t = 3?

$v(2) = 18$ $s(0) = 8$

$v(t) = 6t^2 - 4t + C$

@$(2, 18)$ $18 = 6(2)^2 - 4(2) + C$

$18 = 6(4) - 8 + C$

$18 = 24 - 8 + C$

$18 = 16 + C$

$2 = C$

$\therefore v(t) = 6t^2 - 4t + 2$

$s(0) = 8$

$s(t) = 2t^3 - 2t^2 + 2t + C$

@$(0, 8)$ $8 = 2(0)^3 - 2(0)^2 + 2(0) + C$

$8 = C$

$\therefore s(t) = 2t^3 - 2t^2 + 2t + 8$

@ $t = 3$

$x(3) = 2(3)^3 - 2(3)^2 + 2(3) + 8$

$= 2(27) - 2(9) + 6 + 8$

$= 54 - 18 + 14$

$= 54 - 4$

$x(3) = 50$ cm

The position of the particle is 50 cm at 3 seconds.