

Basic Integration

8.3 – Antiderivatives

Find the antiderivative.

$$\#1) f'(x) = \sqrt[5]{x^2} + \frac{4}{\sqrt[4]{x^3}} = x^{2/5} + 4x^{-3/4}$$

$$f(x) = \frac{5}{7}x^{7/5} + 4(4)x^{-1/4}$$

$$f(x) = \frac{5}{7}\sqrt[5]{x^7} + 16\sqrt[4]{x}$$

Recall... each derivative

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\sin x) = \cos x$$

More Antiderivative formulas

$$\int \frac{1}{x} dx = \ln|x| + C \quad \int e^x = e^x + C$$

$$\int \cos x dx = \sin x + C \quad \int \sin x dx = -\sin x + C$$

You can calculate C if you're given a condition.

Find the function f that satisfies the given condition.

$$\#1) f'(x) = 3x^2 - x + 4 \text{ and } f(2) = 8$$

$$f(x) = x^3 - \frac{1}{2}x^2 + 4x + C$$

$$\textcircled{2.8}: 8 = (2)^3 - \frac{1}{2}(2)^2 + 4(2) + C$$

$$8 = 8 - \frac{1}{2}(4) + 8 + C$$

$$8 = 16 - 2 + C$$

$$8 = 14 + C$$

$$-6 = C$$

$$\therefore f(x) = x^3 - \frac{1}{2}x^2 + 4x - 6$$

$$\#2) f'(x) = \frac{1}{x} - e^x \text{ and } f(1) = e$$

$$f(x) = \ln|x| - e^x + C$$

$$\textcircled{1.e}: e = \ln|1| - e^1 + C$$

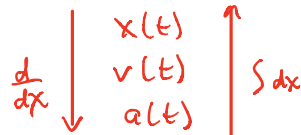
$$e = 0 - e + C$$

$$2e = C$$

$$\therefore f(x) = \ln|x| - e^x + 2e$$

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8.3 – Antiderivatives



#3) Find y if $y'' = \cos x$ and $y'(\frac{\pi}{2}) = 2$ and $y(\frac{\pi}{2}) = 3\pi$

$$y' = \sin x + C$$

$$\textcircled{(\frac{\pi}{2}, 2)}: 2 = \sin \frac{\pi}{2} + C$$

$$2 = 1 + C$$

$$1 = C$$

$$\therefore y' = \sin x + 1$$

$$y = -\cos x + x + C$$

$$\textcircled{(\frac{\pi}{2}, 3\pi)}: 3\pi = -\cos \frac{\pi}{2} + \frac{\pi}{2} + C$$

$$3\pi = 0 + \frac{\pi}{2} + C$$

$$\frac{6\pi}{2} = \frac{\pi}{2} + C$$

$$\frac{5\pi}{2} = C$$

$$\therefore y = -\cos x + x + \frac{5\pi}{2}$$

Particle Motion

A particle moves along the x -axis with an acceleration of $a(t) = 12t - 4$. The particle's velocity is 18 centimeters per second at $t = 2$. The initial position of the particle is 8 cm. What is the position of the particle at $t = 3$?

$$v(2) = 18$$

$$s(0) = 8$$

$$v(t) = 6t^2 - 4t + C$$

$$\textcircled{(2, 18)}: 18 = 6(2)^2 - 4(2) + C$$

$$18 = 6(4) - 8 + C$$

$$18 = 24 - 8 + C$$

$$18 = 16 + C$$

$$2 = C$$

$$\therefore v(t) = 6t^2 - 4t + 2$$

$$s(0) = 8$$

$$s(t) = 2t^3 - 2t^2 + 2t + C$$

$$\textcircled{(0, 8)}: 8 = 2(0)^3 - 2(0)^2 + 2(0) + C$$

$$8 = C$$

$$\therefore s(t) = 2t^3 - 2t^2 + 2t + 8$$

$$\textcircled{t=3}$$

$$x(3) = 2(3)^3 - 2(3)^2 + 2(3) + 8$$

$$= 2(27) - 2(9) + 6 + 8$$

$$= 54 - 18 + 14$$

$$= 54 - 4$$

$$x(3) = 50 \text{ cm}$$

The position of the particle is 50 cm at 3 seconds.