Calculus

9.1 The 2nd Fundamental Theorem of Calculus

Name:_____

Write your questions and thoughts here!

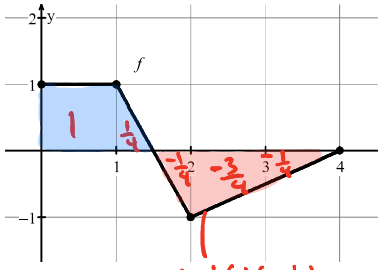
Notes

Function Defined by an Integral:

$$F(x) = \int_0^x f(t)\,dt$$

any #

1. Use the function f in the figure and the function F defined by $F(x) = \int_0^x f(t)\,dt$ on the interval $0 \le x \le 4$.



$A = \frac{1}{2}(1)(1+\frac{1}{2})$
$A = \frac{1}{2}(\frac{3}{2})$
$A = \frac{3}{4}$

a) Complete the table.

x	0	1	2	3	4
$F(x)$	0	1	1	$\frac{1}{4}$	0

$F(x) = \int_0^x f(t)\,dt$

b) When does $F(x)$ have a minimum?

At $x = 0$ and $x = 4$

c) When does $F(x)$ have a maximum?

At $x = 1.5$

d) Integrate $f(t)$.

$$\int_0^x f(t)\,dx$$
$$= F(t)\Big|_0^x$$
$$= F(x) - F(0)$$

↑ Constant

e) Now take the derivative.

$$\frac{d}{dx}\left(F(x) - F(0)\right)$$

↑ Constant

$$= f(x) - 0$$
$$= f(x)$$

Second Fundamental Theorem of Calculus

If $F(x) = \int_a^x f(t)\,dt$, where a is constant and f is a continuous function, then

$$F'(x) = f(x)$$

If $F(x) = \int_a^{g(x)} f(t)\,dt$, where a is constant, f is a continuous function, and g is a differentiable function, then Chain Rule

$$F'(x) = f(g(x)) \cdot g'(x)$$

9.1 The 2nd FTC

Notes

Write your questions and thoughts here!

Find $F'(x)$.

1. $F(x) = \int_2^x (3t^2 + 4t)\,dt$

$F'(x) = 3x^2 + 4x$

2. $F(x) = \int_{\pi/2}^{x^3} \sin(t)\,dt$

$F'(x) = \sin(x^3)\cdot(x^3)'$

$= 3x^2 \sin(x^3)$

3. $F(x) = \int_1^{4x} h(t)\,dt$

$F'(x) = h(4x)\cdot(4x)'$

$= 4h(4x)$

4. $F(x) = \int_{-x}^{x} 5t\,dt$

$F(x) = \int_{-x}^{0} 5t\,dt + \int_{0}^{x} 5t\,dt$

$= -\int_{0}^{-x} 5t\,dt + \int_{0}^{x} 5t\,dt$

$F'(x) = -5(-x)\cdot(-x)' + 5(x)\cdot x'$

$= 5x(-1) + 5(x)\cdot 1$

$F(x) = 0$

5. $F(x) = \int_{2x}^{3x} (t^2 - t)\,dt$

$F(x) = \int_{2x}^{0} (t^2-t)\,dt + \int_{0}^{3x} (t^2-t)\,dt$

$F(x) = -\int_{0}^{2x} (t^2-t)\,dt + \int_{0}^{3x} (t^2-t)\,dt$

$F'(x) = -\left[(2x)^2 - (2x)\right]\cdot(2x)' + \left[(3x)^2 - (3x)\right](3x)'$

$= (-4x^2 + 2x)\cdot 2 + (9x^2 - 3x)\cdot 3$

$= -8x^2 + 4x + 27x^2 - 9x$

$F'(x) = 19x^2 - 5x$

Now summarize what you learned!