

9.2 Trig Integrals

Name: _____

Write your questions
and thoughts here!

Notes

Recall: What are the six trig derivatives? **GIVEN u is a function.**

$$\frac{d}{dy} \sin(u) = \cos(u) \cdot u'$$

$$\frac{d}{dy} \csc(u) = -\csc(u) \cot(u) \cdot u'$$

$$\frac{d}{dy} \cos(u) = -\sin(u) \cdot u'$$

$$\frac{d}{dy} \sec(u) = \sec(u) \tan(u) \cdot u'$$

$$\frac{d}{dy} \tan(u) = \sec^2(u) \cdot u'$$

$$\frac{d}{dy} \cot(u) = -\csc^2(u) \cdot u'$$

Trig Integrals:

$$\int \cos x \, dx = \sin x + C$$

$$\int -\csc x \cot x \, dx = \csc x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

Preparing for u -substitution: *Reverse Chain Rule*

$$\int \cos ax \, dx = \frac{1}{a} \sin(ax) + C$$
only for linear

Find the indefinite integral.

1. $\int -5 \sin x \, dx$

$$\begin{aligned} &= -5 \int \sin x \, dx \\ &= -5 (-\cos x) + C \\ &= 5 \cos x + C \end{aligned}$$

2. $\int \frac{2}{\sec x} \, dx = 2 \int \cos x \, dx$

$$= 2 \sin x + C$$

Evaluate each definite integral.

3. $\int_{\pi/4}^{\pi} -2 \cos x \, dx$

$$\begin{aligned} &= -2 \int_{\pi/4}^{\pi} \cos x \, dx \\ &= -2 \sin x \Big|_{\pi/4}^{\pi} \\ &= -2 \left[\sin \pi - \sin \frac{\pi}{4} \right] \\ &= -2 \left[0 - \frac{\sqrt{2}}{2} \right] \\ &= -2 \left[-\frac{\sqrt{2}}{2} \right] \\ &= \sqrt{2} \end{aligned}$$

4. $\int_{\pi/4}^{3\pi/4} \sec^2 2x \, dx$

$$\begin{aligned} &= \frac{1}{2} \tan(2x) \Big|_{\pi/4}^{3\pi/4} \\ &= \frac{1}{2} \left[\tan\left(2 \cdot \frac{3\pi}{4}\right) - \tan\left(2 \cdot \frac{\pi}{4}\right) \right] \\ &= \frac{1}{2} \left[\tan\left(\frac{3\pi}{2}\right) - \tan\left(\frac{\pi}{2}\right) \right] \\ &= \frac{1}{2} \left[\text{und} - \text{und} \right] \\ &= \text{undefined} \end{aligned}$$

5. $\int_{-\pi/16}^0 \sec 4x \tan 4x \, dx$

$$\begin{aligned} &= \frac{1}{4} \sec(4x) \Big|_{-\pi/16}^0 \\ &= \frac{1}{4} \left[\sec(4(0)) - \sec\left(4\left(-\frac{\pi}{16}\right)\right) \right] \\ &= \frac{1}{4} \left[\sec(0) - \sec\left(-\frac{\pi}{4}\right) \right] \\ &= \frac{1}{4} \left[1 - \sqrt{2} \right] \\ &= \frac{1}{4} - \frac{\sqrt{2}}{4} \\ &= \frac{1-\sqrt{2}}{4} \end{aligned}$$

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Recall the inverse trig derivatives. Remember that arcsine(x) is the same as $\sin^{-1} x$.

Inverse Trig Derivatives: GIVEN u is a function.

$$\frac{d}{dx} \sin^{-1}(u) = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} \cos^{-1}(u) = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} \sec^{-1}(u) = \frac{u'}{|u| \sqrt{u^2-1}}$$

$$\frac{d}{dx} \csc^{-1}(u) = \frac{-u'}{|u| \sqrt{u^2-1}}$$

$$\frac{d}{dx} \tan^{-1}(u) = \frac{u'}{u^2+1}$$

$$\frac{d}{dx} \cot^{-1}(u) = \frac{-u'}{u^2+1}$$

Taking the integral is just going the other direction!

Find the indefinite integral.

$$6. \int -\frac{1}{\sqrt{1-x^2}} dx = \cos^{-1}(x) + C$$

$$7. \int \frac{3}{9x^2+1} dx = \tan^{-1}(3x) + C$$

$$8. \int -\frac{1}{|x|\sqrt{4x^2-1}} dx = \csc^{-1}(2x) + C$$

$$9. \int \frac{20x^3}{\sqrt{1-25x^8}} dx = \sin^{-1}(5x^4) + C$$

Now
summarize
what you
learned!
