Fundamental Theorem of Calculus Part 2

9.3 – Average Rate of Change

Notes

Recall:

Average Rate of Change: $\text{Slope} = \frac{f(b)-f(a)}{b-a}$

Mean Value Theorem (MVT) for Derivatives: $f'(c) = \frac{f(b)-f(a)}{b-a}$ for some c value.

Average Value of a Function:

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

1. Find the average value of $f(x) = 6 - x^2$ on $[-1, 3]$.

$$Av = \frac{1}{3-(-1)}\int_{-1}^{3}(6-x^2)dx = \frac{1}{4}\left(6x - \frac{1}{3}x^3\right)\Big|_{-1}^{3}$$

$$= \frac{1}{4}\left(\left[6(3) - \frac{1}{3}(3)^3\right] - \left[6(-1) - \frac{1}{3}(-1)^3\right]\right)$$

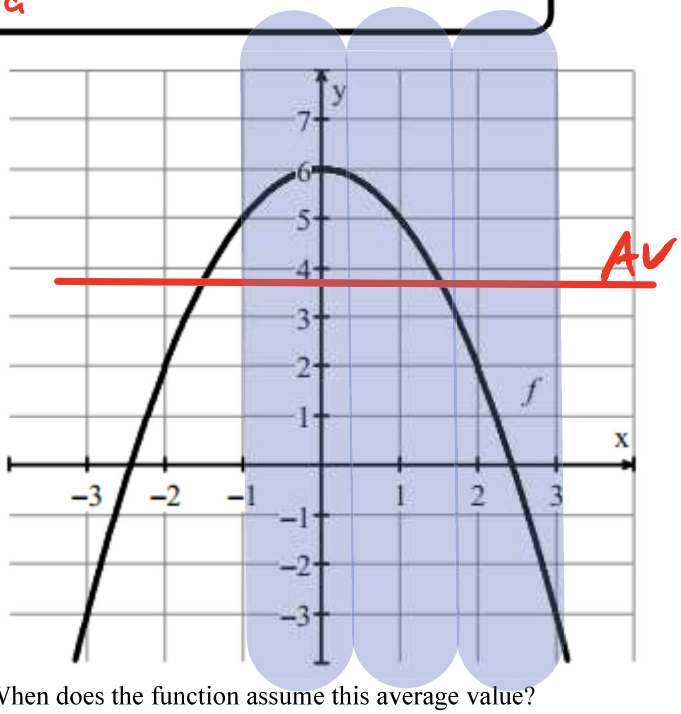
$$= \frac{1}{4}\left(\left[18 - 9\right] - \left[-6 + \frac{1}{3}\right]\right)$$

$$= \frac{1}{4}\left(\left[9\right] - \left[-\frac{18}{3} + \frac{1}{3}\right]\right)$$

$$= \frac{1}{4}\left[\frac{27}{3} + \frac{17}{3}\right]$$

$$= \frac{1}{4}\left[\frac{44}{3}\right]$$

$$Av = \frac{11}{3}$$



When does the function assume this average value?

$$\frac{11}{3} = 6 - x^2$$

$$\frac{11}{3} - \frac{18}{3} = -x^2$$

$$-\frac{7}{3} = -x^2$$

$$\frac{7}{3} = x^2$$

$$\pm\sqrt{\frac{7}{3}} = x$$

$$1.528 \approx x$$

Fundamental Theorem of Calculus Part 2

9.3 – Average Rate of Change

MVT for Integrals:

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Comparing ***average rate of change*** (secant slope) and ***average value*** of a function.

Set up the equation for each question, but do not solve it. What units will the ANSWER be?

2. $h(t) = -16t^2 + 41t + 10$. h is height (feet) and t is time (seconds).
   a. What is the average height during the first 3 seconds?

$$AH = \frac{1}{3}\int_0^3 h(t)\,dt \quad \text{in feet}$$

   b. What is the average velocity during the first 3 seconds?

$$ARC = \frac{h(3)-h(0)}{3} \quad \text{ft/sec}$$

3. $r(x) = 2\sin x - 1$, where r is the rate at which Mr. Brust's waistline is changing (inches per month) and x is time (months).
   a. What is the average rate that Mr. Brust's waistline changes from the 10th to the 12th month?

$$A\ rate = \frac{1}{12-10}\int_{10}^{12} r(x)\,dx \quad \text{in/month}$$

   b. What is the average change of this rate during the first 5 months?

$$ARC = \frac{r(5)-r(0)}{5} \quad \text{in/month}^2$$