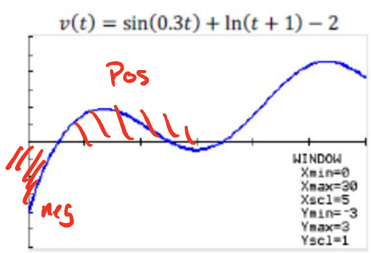
Basic Integration

9.4 – Net Change

When you integRATE a RATE, you get net change.

$$\int \text{rate of change}\, dt = \text{net change}$$

#1) George is driving across town to Danny Devito's house to play with a new set of Teenage Mutant Ninja Turtles. George's speed would obviously vary throughout the drive, but because he is so cool he came up with a function that represents his velocity (miles per minute) at any given time t (minutes) since he left his house during the 30 minute drive.



Set up the expressions for the following scenarios. Use a calculator to solve.

a. How far is George from his house after 10 minutes? Is he going away from his house or towards his house at that moment?

$$\int_0^{10} v(t)\,dt \approx 3.010 \text{ miles}$$

b. How far is George from his house after 15 minutes? Is he going away from his house or towards his house at that moment?

$$\int_0^{15} v(t)\,dt \approx 3.397 \text{ miles}$$

c. If George arrives at Devito's house after 30 minutes, how far away does he live?

$$\int_0^{30} v(t)\,dt = 22.824 \text{ miles}$$

d. How many miles did George drive?

$$\int_0^{30} \text{ABS}(v(t))\,dt \approx 28.497 \text{ miles}$$

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9.4 – Net Change

$\int$ velocity $dt =$ displacement

$\int |\text{velocity}|\, dt =$ total distance

Don't get this confused with: |velocity| = speed

#2) The function $v(t) = t^3 - 2t^2 + 1$ is a particle's velocity.
If $x(t)$ represents the position of the particle along the x-axis, find the following:

a. The position of the particle after 3 seconds if $x(0) = 5$.

$$5 + \int_0^3 (t^3 - 2t^2 + 1)\,dt = 5 + \left(\tfrac{1}{4}t^4 - \tfrac{2}{3}t^3 + t\right)\Big|_0^3$$

$$= 5 + \left[\tfrac{1}{4}(3)^4 - \tfrac{2}{3}(3)^3 + (3)\right] - \left[\tfrac{1}{4}(0)^4 - \tfrac{2}{3}(0)^3 + (0)\right]$$

$$= 5 + \left[\tfrac{81}{4} - \tfrac{2}{3}(27) + 3\right] - [0]$$

$$= 5 + \left[\tfrac{81}{4} - 18 + 3\right]$$

$$= \tfrac{81}{4} - 10$$

$$= \tfrac{81}{4} - \tfrac{40}{4}$$

$$= \tfrac{41}{4}$$

b. The position of the particle after 2 seconds if $x(1) = -2$

$$-2 + \int_1^2 (t^3 - 2t^2 + 1)\,dt = -2 + \left(\tfrac{1}{4}t^4 - \tfrac{2}{3}t^3 + t\right)\Big|_1^2$$

$$= -2 + \left[\tfrac{1}{4}(2)^4 - \tfrac{2}{3}(2)^3 + (2)\right] - \left[\tfrac{1}{4}(1)^4 - \tfrac{2}{3}(1)^3 + (1)\right]$$

$$= -2 + \left[\tfrac{1}{4}(16) - \tfrac{2}{3}(8) + 2\right] - \left[\tfrac{1}{4}(1) - \tfrac{2}{3}(1) + 1\right]$$

$$= -2 + \left[4 - \tfrac{16}{3} + 2\right] - \left[\tfrac{1}{4} - \tfrac{2}{3} + 1\right]$$

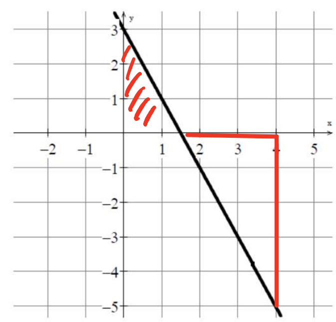
$$= -2 + 6 - \tfrac{16}{3} - \tfrac{1}{4} + \tfrac{2}{3} - 1$$

$$= 3 - \tfrac{14}{3} - \tfrac{1}{4}$$

$$= \tfrac{36}{12} - \tfrac{56}{12} - \tfrac{3}{12}$$

$$= -\tfrac{23}{12}$$

#3) What is the total distance traveled by a particle during the first 4 seconds if the particle's velocity function is given by $v(t) = -2t + 3$? Show the set up AND your work.



$$3 + \int_0^4 |-2t + 3|\,dt$$

$$= 3 + \int_0^{1.5} |-2t + 3|\,dt + \int_{1.5}^4 |-2t + 3|\,dt$$

$$= 3 + \left(\tfrac{1}{2} \cdot \tfrac{3}{2} \cdot 3\right) + \left(\tfrac{1}{2} \cdot \tfrac{5}{2} \cdot 5\right)$$

$$= \tfrac{12}{4} + \tfrac{9}{4} + \tfrac{25}{4}$$

$$= \tfrac{46}{4}$$

$$= \tfrac{23}{2}$$

#4) If $H(-\pi) = 12$ and $H'(t) = \cos(t)$, what is $H\left(\frac{3\pi}{2}\right)$?

$$H = \int \cos(t)\,dt$$

$$H = \sin(t) + C$$

@ $(-\pi, 12)$

$$12 = \sin(-\pi) + C$$

$$12 = 0 + C$$

$$12 = C$$

$$H = \sin(t) + 12$$

$$H\left(\tfrac{3\pi}{2}\right) = \sin\left(\tfrac{3\pi}{2}\right) + 12$$

$$= -1 + 12$$

$$H\left(\tfrac{3\pi}{2}\right) = 11$$