

Fundamental Theorem of Calculus Part 2

Review 9 – Smac Edition

Find $F'(x)$.

$$1. F(x) = \int_4^x \frac{1}{\sqrt{t}} dt$$

$$F'(x) = \frac{1}{\sqrt{x}}$$

$$2. F(x) = \int_x^3 t^2 dt$$

$$F(x) = -\int_x^3 t^2 dt$$

$$F'(x) = -x^2$$

$$3. F(x) = \int_{\pi}^x \tan t dt$$

$$F'(x) = \tan x$$

$$4. F(x) = \int_x^{5x} \frac{1}{t} dt = -\int_x^{\cancel{x}} \frac{1}{t} dt$$

$$F'(x) = -\frac{1}{x}$$

$$5. F(x) = \int_{-1}^{2x} (1-t^2) dt$$

$$F'(x) = (2x)' \cdot (1-(2x)^2)$$

$$F'(x) = 2(1-4x^2)$$

$$F'(x) = 2-8x^2$$

$$6. F(x) = \int_e^x \ln t dt$$

$$F'(x) = (e^x)' \ln e^x$$

$$= e^x \cdot x \ln e$$

$$F'(x) = x e^x$$

$$7. F(x) = \int_9^{x^4} \sqrt{t} dt$$

$$F'(x) = (x^4)' \sqrt{x^4}$$

$$= 4x^3 x^2$$

$$F'(x) = 4x^5$$

$$8. F(x) = \int_0^{x^2-x} t^2 dt$$

$$F'(x) = (x^2-x)' (x^2-x)^2$$

$$F'(x) = (2x-1)(x^2-x)^2$$

$$9. F(x) = \int_{-\pi}^{\cos x} 2^t dt$$

$$F'(x) = (\cos x)' 2^{\cos x}$$

$$= -\sin x \cdot 2^{\cos x}$$

$$F'(x) = -\frac{\cos x}{\sin x}$$

$$10. F(x) = \int_{-x}^x \sin^2 t dt$$

$$F(x) = \int_{-x}^0 \sin^2 t dt + \int_0^x \sin^2 t dt$$

$$F(x) = -\int_0^{-x} \sin^2 t dt + \int_0^x \sin^2 t dt$$

$$F'(x) = (-x)' (-\sin^2(-x)) + \sin^2(x)$$

$$= -(-\sin^2(x)) + \sin^2 x$$

$$F'(x) = \sin^2(-x) + \sin^2 x$$

$$11. F(x) = \int_{-x}^{3x^2} t^2 dt$$

$$F(x) = \int_{-x}^0 t^2 dt + \int_0^{3x^2} t^2 dt$$

$$F(x) = -\int_0^{-x} t^2 dt + \int_0^{3x^2} t^2 dt$$

$$F'(x) = (-x)' (-x^2) + (3x^2)' (3x^2)^2$$

$$F'(x) = (-1)(-x^2) + 6x(9x^4)$$

$$F'(x) = x^2 + 54x^5$$

$$F'(x) = 54x^5 + x^2$$

$$12. F(x) = \int_{x^2}^{x^4} \sqrt{t} dt$$

$$F(x) = \int_{x^2}^0 \sqrt{t} dt + \int_0^{x^4} \sqrt{t} dt$$

$$F(x) = -\int_0^{x^2} \sqrt{t} dt + \int_0^{x^4} \sqrt{t} dt$$

$$F'(x) = (x^2)' (-\sqrt{x^2}) + (x^4)' (\sqrt{x^4})$$

$$= 2x(-x) + 4x^3(x^2)$$

$$= -2x^2 + 4x^5$$

$$F'(x) = 4x^5 - 2x^2$$

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Find each integral.

#13) $\int (12x^3 + 3x^2 - 5)dx$

$$= 3x^4 + x^3 - 5x + C$$

#14) $\int 9 \cos(x) \tan(x) dx$

$$\begin{aligned} &= 9 \int \cancel{\cos(x)} \cdot \frac{\sin(x)}{\cancel{\cos(x)}} dx \\ &= 9 \int \sin(x) dx \\ &= -9 \cos(x) + C \end{aligned}$$

#15) $\int \frac{x^2 - 1}{x-1} dx$

$$\begin{aligned} &= \int \frac{(x-1)(x+1)}{x-1} dx \\ &= \int (x+1) dx \\ &= \frac{1}{2}x^2 + x + C \end{aligned}$$

#16) $\int \left[\frac{\sec(x)}{\cos(x)} - \frac{\tan(x)}{\cot(x)} \right] dx$

$$\begin{aligned} &= \int \left(\frac{\sec(x)}{\frac{1}{\sec(x)}} - \frac{\tan(x)}{\frac{1}{\tan(x)}} \right) dx \\ &= \int [\sec^2(x) - \tan^2(x)] dx \\ &= \int [1 + \tan^2(x) - 1 - \tan^2(x)] dx \\ &= \int 1 dx \\ &= x + C \end{aligned}$$

#17) $\int \sqrt{(\csc(x) - 1)(\csc(x) + 1)} dx$

$$\begin{aligned} &= \int \sqrt{\csc^2(x) - 1} dx \\ &= \int \sqrt{\cot^2(x)} dx \\ &= \int |\cot(x)| dx \\ &= \ln|\sin(x)| + C \end{aligned}$$

#18) $\int (x^2 + x + 1 + x^{-1} + x^{-2}) dx$

$$= \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \ln|x| - x^{-1} + C$$

#19) $\int (6e^{\frac{2x}{3}}) dx$

$$\begin{aligned} &= 6\left(\frac{3}{2}\right)e^{\frac{2x}{3}} + C \\ &= 9e^{\frac{2x}{3}} + C \end{aligned}$$

#20) $\int (e^{3x} - \frac{3}{x}) dx$

$$= \frac{1}{3}e^{3x} - 3\ln|x| + C$$

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#21) A company's marginal cost function is $MC(x) = 21x^{4/3} - 6x^{\frac{1}{2}} + 50$, where x is the number of units, and fixed costs are \$3000. Find the cost function.

$$\boxed{x = \text{# of units}}$$

$$C(x) = \text{Total Cost}$$

$$C(x) = \int (21x^{\frac{4}{3}} - 6x^{\frac{1}{2}} + 50) dx$$

$$= 21\left(\frac{1}{7}\right)x^{\frac{7}{3}} - 6\left(\frac{2}{3}\right)x^{\frac{3}{2}} + 50x + C$$

$$C(x) = 9x^{\frac{7}{3}} - 4x^{\frac{3}{2}} + 50x + C$$

$$3000 = 9(0)^{\frac{7}{3}} - 4(0)^{\frac{3}{2}} + 50(0) + C$$

$$3000 = C$$

$$C(x) = 9x^{\frac{7}{3}} - 4x^{\frac{3}{2}} + 50x + 3000$$

#22) A factory installs new equipment that is expected to generate savings at the rate of $800e^{-0.2t}$ dollars per year, where t is the number of years that the equipment has been in operation.

- a. Find a formula for the total savings that the equipment will generate during its first t years.
 $\boxed{(0, 0)}$
- b. If the equipment originally cost \$2000, when will it "pay for itself"?
 $\boxed{t = \text{years}}$

$$a. T = \int 800e^{-0.2t} dt$$

$$= 800(s)e^{-0.2t} + C$$

$$T = -4000e^{-0.2t} + C$$

$$0 = -4000e^{-0.2(0)} + C$$

$$0 = -4000e^0 + C$$

$$0 = -4000(1) + C$$

$$4000 = C$$

$$T = -4000e^{-0.2t} + 4000$$

b.

$$2000 = -4000e^{-0.2t} + 4000$$

$$-2000 = -4000e^{-0.2t}$$

$$\frac{1}{2} = e^{-0.2t}$$

$$\ln\left(\frac{1}{2}\right) = \ln(e^{-0.2t})$$

$$\ln\left(\frac{1}{2}\right) = -0.2t$$

$$-5\ln\left(\frac{1}{2}\right) = t$$

$$3.5 \approx t$$

The equipment will pay for itself in 3.5 years

#23) A flu epidemic hits a college community, beginning with five cases on day $t = 0$. The rate of growth of the epidemic (new cases per day) is given by $r(t) = 18e^{0.05t}$, where t is the number of days since the epidemic began.

- Find a formula for the total number of cases of flu in the first t days.
- Use your answer to part (a) to find the total number of cases in the first 20 days.

$$(0, 5)$$

$$\boxed{F = \text{total flu cases}}$$

$$\boxed{t = \text{days}}$$

$$a. F = \int 18e^{0.05t} dt$$

$$= 18(20)e^{0.05t} + C$$

$$F = 360e^{0.05t} + C$$

$$S = 360e^{0.05(0)} + C$$

$$S = 360e^0 + C$$

$$S = 360(1) + C$$

$$-355 = C$$

$$F = 360e^{0.05t} - 355$$

b.

$$F(20) = 360e^{0.05(20)} - 355$$

$$= 360e^1 - 355$$

$$F(20) \approx 624 \text{ cases}$$

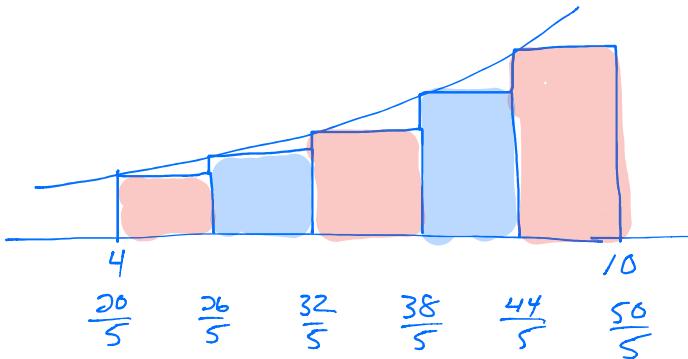
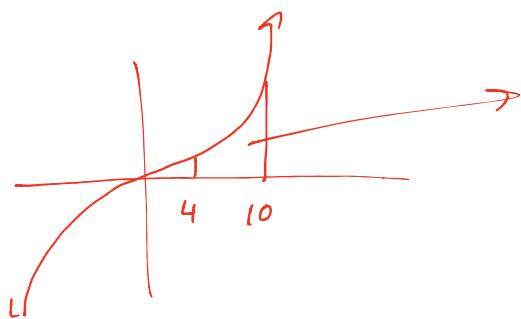
There will be 624 flu cases during the first 20 days.

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Find the area under the curve using 5 rectangles. Draw a sketch of the area

#24) $f(x) = x^3$ from $x = 4$ to $x = 10$



$$\Delta x = \frac{b-a}{n} = \frac{10-4}{5} = \frac{6}{5}$$

$$A = \int_{4}^{10} x^3 dx$$

$$\approx f(x_1) \Delta x + f(x_2) \Delta x + f(x_3) \Delta x + f(x_4) \cdot \Delta x + f(x_5) \cdot \Delta x$$

$$\approx f(4) \cdot \frac{6}{5} + f\left(\frac{26}{5}\right) \cdot \frac{6}{5} + f\left(\frac{32}{5}\right) \cdot \frac{6}{5} + f\left(\frac{38}{5}\right) \cdot \frac{6}{5} + f\left(\frac{44}{5}\right) \cdot \frac{6}{5}$$

$$\approx \frac{6}{5} \left[(4)^3 + \left(\frac{26}{5}\right)^3 + \left(\frac{32}{5}\right)^3 + \left(\frac{38}{5}\right)^3 + \left(\frac{44}{5}\right)^3 \right]$$

$$\approx \frac{6}{5} \left[\frac{8000}{125} + \frac{17576}{125} + \frac{32768}{125} + \frac{54872}{125} + \frac{85184}{125} \right]$$

$$\approx \frac{6}{5} \left[\frac{198400}{125} \right]$$

$$\approx \frac{1190400}{625}$$

$$A \approx 1904.64 \text{ un}^2$$

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#25) Explain how a Riemann Sum is used to calculate integrals.

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Use the Fundamental Theorem of Calculus to find the area under the curve.

#26) $f(x) = \frac{\tan(x)\cos(x)}{\sin(x)}$ from $x = 4$ to $x = 10$

$$A = \int_4^{10} \frac{\frac{\sin(x)}{\cos(x)} \cdot \cos(x)}{\sin(x)} dx = \int_4^{10} \frac{1}{\sin(x)} dx$$

$$= \int_4^{10} 1 dx = x \Big|_4^{10} = (10) - (4)$$

$$A = 6 \text{ un}^2$$

#27) $f(x) = x$ from $x = 0$ to $x = 4$

$$A = \int_0^4 x dx$$

$$= \frac{1}{2}x^2 \Big|_0^4$$

$$= \frac{1}{2}(4)^2 - \frac{1}{2}(0)^2$$

$$= \frac{1}{2}(16) - \frac{1}{2}(0)$$

$$= 8 - 0$$

$$A = 8 \text{ un}^2$$

#28) $f(x) = e^x$ from $x = 0$ to $x = 1$

$$A = \int_0^1 e^x dx$$

$$= e^x \Big|_0^1$$

$$= e^{(1)} - e^{(0)}$$

$$A = (e - 1) \text{ un}^2$$

#29) $f(x) = 9 - 3\sqrt{x}$ from $x = 0$ to $x = 9$.

$$A = \int_0^9 (9 - 3x^{\frac{1}{2}}) dx$$

$$= [9x - 2x^{\frac{3}{2}}] \Big|_0^9$$

$$= [9(9) - 2(9)^{\frac{3}{2}}] - [9(0) - 2(0)^{\frac{3}{2}}]$$

$$= [81 - 2(27)] - [0 - 0]$$

$$= [81 - 54] - [0]$$

$$= 27 - 54$$

$$A = 27 \text{ un}^2$$

#30) $f(x) = \frac{1}{x}$ from $x = e$ to $x = e^3$.

$$A = \int_e^{e^3} \frac{1}{x} dx$$

$$= \ln|x| \Big|_e^{e^3}$$

$$= \ln|e^3| - \ln|e|$$

$$= 3 - 1$$

$$A = 2 \text{ un}^2$$

#31) $\int_0^1 (6x^2 - 4e^{2x}) dx$

$$= (2x^3 - 2e^{2x}) \Big|_0^1$$

$$= [2(1)^3 - 2e^{2(1)}] - [2(0)^3 - 2e^{2(0)}]$$

$$= [2(1) - 2e^2] - [0 - 2e^0]$$

$$= [2 - 2e^2 + 2(1)]$$

$$= (4 - 2e^2) \text{ un}^2$$

#32) $\int_1^2 \frac{(x+1)^2}{x^2} dx$

$$A = \int_1^2 \frac{x^2 + 2x + 1}{x^2} dx$$

$$= \int_1^2 (1 + 2x^{-1} + x^{-2}) dx$$

$$= (x + 2\ln|x| - x^{-1}) \Big|_1^2$$

$$= [(2) + 2\ln|2| - (2)^{-1}] - [(1) + 2\ln|1| - (1)^{-1}]$$

$$= [2 + 2\ln|2| - \frac{1}{2}] - [1 + 2(0) - 1]$$

$$= [1.5 + 2\ln(2)] - [0]$$

$$= [1.5 + 2\ln(2)] \text{ un}^2$$

$$\approx 2.89 \text{ un}^2$$

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#33) An average child of age x years gains weight at the rate of $3.9x^{1/2}$ pounds per year (for the first 16 years). Find the total weight gain from age 1 to age 9.

$$\begin{aligned} x &= \text{age of child} \\ WG &= \text{weight gained} \\ WG &= \int_1^9 3.9x^{1/2} dx = 3.9\left(\frac{2}{3}x^{3/2}\right) \Big|_1^9 \\ &= 2.6(\sqrt{x})^3 \Big|_1^9 \\ &= 2.6(\sqrt{9})^3 - 2.6(\sqrt{1})^3 \\ &= 2.6(3)^3 - 2.6(1) \\ &= 2.6(27) - 2.6 \\ &= 70.2 - 2.6 \\ WG &= 1 \text{ lbs.} \end{aligned}$$

The total weight gained from 1 to 9 is 1 lbs.

#34) A company's marginal cost function is $MC(x) = 8e^{-0.01x}$, where x is the number of units. Find the total cost of the first hundred units. ($x = 0$ to $x = 100$).

$$\begin{aligned} C &= \int_0^{100} 8e^{-0.01x} dx \\ &= 8(-100)e^{-0.01x} \Big|_0^{100} \\ &= (-800e^{-0.01(100)}) - (-800e^{-0.01(0)}) \\ &= -800e^{-1} + 800e^0 \\ &= -800 \cdot \frac{1}{e} + 800 \\ C &\approx \$505.70 \end{aligned}$$

The first 100 units cost about \$505.70

#35) There are approximately $2.3e^{0.01t}$ million marriages per year in the United States, where t is the number of years since 1995. Assuming that this rate continues, find the average number of marriages from the year 1995 to the year 2005.

$$\begin{aligned} \text{AM} &= \frac{1}{10-0} \int_0^{10} 2.3e^{0.01t} dt \\ &= \frac{1}{10} \left[2.3(100)e^{0.01t} \right] \Big|_0^{10} \\ &= \frac{1}{10} \left[230e^{0.01(10)} - 230e^{0.01(0)} \right] \\ &= \frac{1}{10} [230e^{-1} - 230e^0] \\ &= \frac{1}{10} [230e^{-1} - 230] \\ M &\approx 2.418931 \text{ million marriages} \end{aligned}$$

There were about 2,418,931 marriages from 1995 to 2005.

#36) What is the difference between an indefinite integral and a definite integral?

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#37) Explain in words what the answer to this problem means.

$$\int_3^7 x^2 dx$$

(note: you do not need to actually solve this problem.)

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