A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.
You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

5. The rate at which cars enter a parking lot is modeled by $E(t) = 30 + 5(t - 2)(t - 5)e^{-0.2t}$. The rate at which cars leave the parking lot is modeled by the differentiable function L. Selected values of $L(t)$ are given in the table. Both $E(t)$ and $L(t)$ are measured in cars per hour, and time t is measured in hours after 5 A.M. ($t = 0$). Both functions are defined for $0 \le t \le 12$.

	7Am	10Am	2PM	4Pm	5pm
t (hours)	2	5	9	11	12
$L(t)$ (cars per hour)	15	40	24	68	18

(a) What is the rate of change of $E(t)$ at time $t = 7$? Indicate units of measure.

$E'(7) = 6.165 \ \frac{\text{cars per hour}}{\text{hour}}$

The rate of change of E(t) at time t=7 is 6.165 cars per hour per hour. +1

(b) How many cars enter the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole number.

+1

$$\int_0^{12} E(t)\,dt \approx 520.070$$

To the nearest whole number 520 cars enter the parking lot from time $t=0$ to $t=12$ +1

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate $\int_2^{12} L(t)dt$ using correct units, explain the meaning of $\int_2^{12} L(t)dt$ in the context of this problem.

$$\int_2^{12} L(t)dt \approx 3\cdot\tfrac{1}{2}\left[15+40\right] + 4\cdot\tfrac{1}{2}\left[40+24\right] + 2\cdot\tfrac{1}{2}\left[24+68\right] + 1\cdot\tfrac{1}{2}\left[68+18\right] \quad +1$$

$$\approx 345.5 \text{ cars} \quad +1$$

+1

$\int_2^{12} L(t)dt$ represents the number of cars that leave the parking lot from 7AM ($t=2$) to 5PM ($t=12$).

(d) For $0 \leq t < 6$, 5 dollars are collected from each car entering the parking lot. For $6 \leq t \leq 12$, 8 dollars are collected from each car entering the parking lot. How many dollars are collected from the cars entering the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole dollar.

$$\text{Dollars Collected} = 5\int_0^6 E(t)dt + 8\int_6^{12} E(t)dt = 3530.140$$

+1 +1

+1 To the nearest dollar, 3530 dollars were collected from time $t=0$ to time $t=12$.

NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

6. Consider the curve defined by $2x^2 + 3y^2 - 4xy = 36$

(a) Show that $\frac{dy}{dx} = \frac{2y - 2x}{3y - 2x}$

$$\frac{d}{dx}\left(2x^2 + 3y^2 - \overbrace{4xy}^{\text{Product}}\right) = \frac{d}{dx}(36)$$

$$4x + 6y \cdot \frac{dy}{dx} - \underline{4y - 4x\frac{dy}{dx}} = 0$$ +1

$$\frac{dy}{dx}(6y - 4x) = 4y - 4x$$

$$\frac{dy}{dx} = \frac{2(2y-2x)}{2(3y-2x)}$$

$$\frac{dy}{dx} = \frac{2y-2x}{3y-2x}$$ +1

(b) $\boxed{\text{Find the slope}}$ of the <u>line tangent</u> to the curve at each point on the curve <u>where $x = 6$.</u>

$2x^2 + 3y^2 - 4xy = 36$

POT $(6,2)$, $(6,6)$

$x=6$

$2(6)^2 + 3y^2 - 4(6)y = 36$

$72 + 3y^2 - 24y = 36$

$3y^2 - 24y + 36 = 0$

$3(y^2 - 8y + 12) = 0$

$3(y-6)(y-2) = 0$

$y = 6$ or $y = 2$

SOT

$$\left.\frac{dy}{dx}\right|_{(6,2)} = \frac{2(2) - 2(6)}{3(2) - 2(6)} = \frac{-8}{-6} = \frac{4}{3}$$

The slope of the tangent line at point $(6,2)$ is $\frac{4}{3}$. +1

$$\left.\frac{dy}{dx}\right|_{(6,6)} = \frac{2(6) - 2(6)}{3(6) - 2(6)} = \frac{0}{6} = 0$$

The slope of the tangent line at point $(6,6)$ is 0. +1

(c) Find the positive value of x at which the curve has a vertical tangent line. Show the work that leads to your answer,

$$\frac{dy}{dx} = \frac{2y-2x}{3y-2x} \quad \Big| \quad \frac{dy}{dx} \text{ is undefined when } \overbrace{3y-2x=0}^{+1}$$

Solve system of equations

$$\boxed{\begin{array}{r} 2x^2+3y^2-4xy=36 \\ 3y-2x=0 \end{array}} \Longrightarrow y=\frac{2}{3}x$$

$$2x^2+3\left(\frac{2}{3}x\right)^2-4x\left(\frac{2}{3}x\right)=36$$

$$3\cdot\left(2x^2+3\cdot\frac{4}{9}x^2-\frac{8}{3}x^2\right)=36\cdot3$$

$$6x^2+4x^2-8x^2=36\cdot3$$

$$2x^2=36\cdot3$$

$$x^2=\frac{36\cdot3}{2}$$

$$x=\pm\sqrt{\frac{36\cdot3}{2}}$$

+1 $x=\sqrt{\frac{36\cdot3}{2}}$ or $x=\sqrt{54}$

(d) Let x and y be functions of time t that are related to the equation $2x^2+3y^2-4xy=36$. At time $t=1$, the value of x is 2, the value of y is -2, and the value of $\frac{dy}{dx}$ is 4. Find the value of $\frac{dx}{dt}$ at time $t=1$.

+1 $$\frac{d}{dt}\left(2x^2+3y^2-\overbrace{4xy}^{\text{Product}}\right)=\frac{d}{dt}36$$

+1 $$4x\cdot\frac{dx}{dt}+6y\cdot\frac{dy}{dt}-\overbrace{4\frac{dx}{dt}y-4x\cdot\frac{dy}{dt}}=0$$

$x=2$	$t=1$	$y=-2$	$\frac{dy}{dx}=4$

$$4(2)\cdot\frac{dx}{dt}+6(-2)(4)-4\frac{dx}{dt}(-2)-4(2)\cdot4=0$$

$$8\cdot\frac{dx}{dt}-48+8\frac{dx}{dt}-32=0$$

$$16\frac{dx}{dt}=48+32$$

+1 $$\frac{dx}{dt}=\frac{48+32}{16}$$ Answer

or

$$\frac{dx}{dt}=5$$ Answer